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QUANTIFICATION AND UTILIZATION OF SUBJECTIVELY DETERMINED DATA IN THE CONSTRUCTION OF MATHEMATICAL MODELS

A Thesis

By

GRADY LEE HAYNES

Submitted to the Graduate College of the
Texas A&M University in
partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 1966

Major Subject: Computer Science

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Approved as to style and content by:

(Chairman of Committee)

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CHAPTER I

THE STATE OF THE ART

Introduction

The building of models designed to assist in the description of future events has often encountered a very serious limitation. This is due to the methods generally used in developing the mathematical forms included within the model structure. In the general case, the model development has been based upon a form of fitting some type of mathematical function to existing data. The method would then use selected data points to check the mathematical function's capability of describing the process that was being modeled. Estimates of the error or the variance of the estimators could then often be obtained from the existing data in an unbiased manner. It has been recognized by statisticians and other scientists that the relationships derived by this method are generally only applicable over the range of the variables that were used in the derivation of the relationship.

There are two major faults which are inherent in this method of mathematical model development. First, in some areas, such as spacecraft and launch vehicle development, there is very little historical data on which to base a predictor. And second, in planning for efforts of a developmental nature there is generally a necessity for an extrapolation beyond the range of present historical data. In some cases, where either of these two faults has been present, rigid statistical techniques have not been followed, causing these approaches to the development of models to be less than credible. Therefore, it is reasonable to explore a new approach which differs significantly from past efforts, and which can be used to obtain information pertinent to the models that are being applied to management and planning decisions for futuristic processes.

Justification of the Use of Expertise

A quick analysis of past modeling techniques reveals that the problems and faults of these techniques arise mainly from attempts to predict future occurrences with information which is at best current and thus of unknown validity when applied to prediction of future events. With this fact in mind, a model was developed using the only available data on future events: the opinions of experts.

The lack of applications-oriented research in forecasting based upon subjective data may be due in part to the reluctance of some scientists to associate themselves with procedures involving both the dependence upon intuition and the lack of predictability of subjective methods. Others, notably those in the operations research areas, have recognized the fact that the final test of any procedure, whether based upon fact or opinion, is the validity of the results produced by the procedure. From the history of scientific endeavor, it can be seen that precision and formality of procedure are not essential to and not a guarantee of precise results.

Subjective data is one of the most common forms of data used for the decisions of everyday life. These decisions, which determine most of the events in one's life, are of relative unimportance when compared to the major decisions that drastically alter life. For these decisions the average person likely turns to the advice of an expert. Medical, educational, and religious experts are only three of the many types of experts which offer their services and opinions. It is not at all unusual for a person to consult several experts on the same question, using the opinions of all (often weighted relative to their experience bases) to help him reach a decision. This same logic is not unreasonably applied to the area of future event prediction. While limited predictions can be made based upon the record of past statistics in analogous instances, it makes sense to rely on the forecasts of professional experts in the field. They have exhibited the ability to supplement the various explicit elements of the question by appropriate use of their capacities for an intuitive appraisal of the intangible factors.

Previous Uses of Expertise in Forecasting

The building of models based upon expert opinion is not entirely a new concept. To date there have been two major undertakings in this area, namely the Rand Corporation's DELPHI method and the PATTERN method of the Honeywell Corporation, both of which were conducted with the cooperation of the Department of Defense of the United States Government.

The DELPHI Method. The DELPHI method derives its name from "Project DELPHI", which began in the early 1950's at the Rand Corporation. Its objective is the obtaining of the most reliable consensus of a group of experts [1].* It attempts to achieve this by a series of intensive questionnaires interspersed with controlled opinion feedback [2]. The technique involves the repeated questioning of the individual experts, either by interview or questionnaire, and avoids direct confrontation of the experts with one another.

The questions, which are all associated with some central problem, are designed to yield the following information: (1) the reasoning that went into the reply of the respondent to the primary question, (2) the factors he considers relevant to the problem, (3) his own estimate of these factors, and (4) the kind of data that he feels would enable him to arrive at a better appraisal of these factors and, thereby, at a more confident answer to the primary question. The information fed to the experts between rounds of questioning is generally of two kinds, either available data previously requested by one

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*Bracketed numbers refer to references listed on page 85.

or more of the experts, or factors and considerations suggested as potentially relevant by one or another of the respondents. With respect to the latter type of information, an attempt is made to conceal the actual opinion of other respondents and merely to present the factor for consideration without introducing unnecessary bias [3].

This method of controlled interaction among the respondents represents a deliberate attempt to avoid the disadvantages associated with more conventional uses of experts, such as round-table discussions or other forms of confrontation with opposing views. The method employed appears to be more conducive to independent thought on the part of the experts and to aid them in the gradual formation of a considered opinion. The proponents of DELPHI believed that direct confrontation often induces the hasty expression of preconceived notions, an inclination to close one's mind to novel ideas, a tendency to defend a stand once taken, or a predisposition to be swayed by persuasively stated opinions of others.

The DELPHI method was first applied to an attempt to predict the capacity of the United States to withstand a nuclear attack. In this initial test, convergence of the opinions of the experts was attained, and the process was considered a success. Since that time, other experiments using the method have usually attained the desired convergence of opinion, but the convergence has been shown not always to be in the direction of the true answer [4].

PATTERN. The approach used by the Honeywell Corporation in their PATTERN (Planning Assistance Through Technical Evaluation of Relevance Numbers) method is quite different from that of the DELPHI method. Direct contact among the judges is allowed, even encouraged, and the voting is done in the form of a round-table discussion. The developers of the method believe this contributes heavily to a convergence of opinion.

PATTERN was developed with the goal in mind of providing a method for ranking future projects according to their importance in certain specified areas. The experts are asked to give a relative ranking to a number of proposed programs, the opinions are analyzed, and the most important of the programs, in the opinion of the experts, is hopefully identified. After each round of questioning, the results of the round are made known to the judges, and discussions of the results are encouraged. It is felt that in this manner those judges which hold relatively extreme opinions may be confronted with new evidence that might persuade them to change their votes.

The PATTERN procedure has been used several times in ranking the importance of future space programs, and its users have reported that very good results have been obtained. Problems which have arisen thus far have been few, but among them is the influence of domineering personalities in forcing convergence [5]. The DELPHI method recognizes and attempts to remedy this situation.

Improvement of Existing Methods

The previously discussed methods represent two of the possible approaches to the solution of the problems of prediction of future events. However, a new method which attempts to reduce the errors inherent in extrapolation beyond the range of the data is desirable, since the area beyond this range is generally the area of interest.

CHAPTER II

A NEW METHOD

Introduction

The method to be presented in this paper resembles past efforts mainly in that it uses subjectively determined data as a basis for decisions. Of the two major attempts at using this form of data, namely DELPHI and PATTERN, the method more closely resembles the former, borrowing from it the technique of non-confrontation of the judges. Whereas the DELPHI method used repeated rounds of questioning to force convergence of the judges' opinions, no attempt is made in this method to persuade a judge to alter his original opinion, and the final results produced by the method come from a weighted combination of these original opinions. Resemblance to PATTERN comes from the fact that, while not attempting to rank, the judges are asked to choose one answer from a group of possible answers as being the best, or in a sense, the most important answer to a specified question. Variance estimates are made, allowing the calculation of confidence bands around the estimates. This differs from the DELPHI method, where the statistical analysis stops after calculation of the mode, median, and interquartile range of the numbers associated with the respondents' opinions.

The experiment to be discussed is one in which the new model was utilized in an attempt to predict cost-time functions for the various cost categories involved in future space programs. The source of expertise was the Manned Spacecraft Center at Houston, Texas.

Collection of Data

The first phase of this method involves the collection of data. As has been previously pointed out, data consists of the opinions of several experts as to the best answer for some specified question, the question in this case being that of choosing a curve as the one best representative of the cost-time function for the cost category under consideration.

The researchers at both the Honeywell Corporation and the Rand Corporation have noted that the accuracy of the results produced by their methods, PATTERN and DELPHI respectively, are a function of the experience bases of the experts who participate in the experiment. For this reason they have strongly suggested that great care be taken in the choosing of a panel of experts.

There are several criteria for the selection of experts. The first and most obvious of these is a knowledge of the subject under consideration. An expert is utilized because his information and the body of experience at his disposal are expected to insure that he will be able to select the needed items of his background information, determine the character and extent of their relevance, and apply these insights to the formulation of the required personal probability judgments.

An expert's knowledge is not enough; he must be able to put his knowledge to effective use on the predictive problem in hand, and not every expert is able to do this. It becomes necessary to place some check upon the effects of his predictive powers and to take a critical look at his past record of predictive performance [6] .

The simplest way in which to score an expert's performance is in terms of "reliability;" his degree of reliability is the relative frequency of cases in which, when confronted with several alternative hypotheses, he attaches to the eventually correct alternative among them a greater probability than to the others. In cases where some type of record of this performance is kept, his reliability is easily assessed; in other cases, the reliability may be a subjective quantity [7].

Another way to secure a rating of the performance is to ask the expert himself for a self-evaluation of his abilities. This was a part of the original DELPHI method, and the researchers associated with that project reported that the self-appraised competence ratings greatly improved the accuracy of the results derived [8] .

Both of these procedures for weighting of the opinions of the responding judges have been incorporated in the method under discussion. This is an attempt to make the method as unbiased as possible with only one round of questioning.

Since the objective of the application of the method was the prediction of the functional forms of percent cost/percent time relationships in future space programs, the questionnaire submitted to the judges consisted of a group of graphical representations of various functions (see pages 58-73), and the judges were asked to choose one of these functions as best representing his idea of the percent cost/percent time relationship in a specified cost category. One of the first problems encountered was the preparation of a set of curves which could suffice as a selection set, a set which would contain enough curves that every judge could find his conception of the functions, but not so many curves that a problem of distinguishability would arise.

The curves in the selection set in this application were, to be meaningful, monotonically increasing functions within the region $0 \leq$ percent cost ≤ 1, $0 \leq$ percent time ≤ 1. To acquire a selection set, a questionnaire, consisting of a blank grid, was sent to each of the judges who was to participate in the main round of questioning, with the request that he sketch his own idea of the function.

From the number of responses and the close similarities among some of them, it was obvious that all could not be used in the selection set for the main round of questioning. Thus, an attempt was made to devise a procedure for establishing the degree of distinguishability between two continuous functions.

Original plans called for a chi-square test to determine differences between the functions. As is well known, visually different functions (The term "visually different" as used in this paper will mean that an expert, when given a curve for evaluation, can say with assurance that this curve differs from one curve or a specified set of curves which he has already seen.) can usually be made to test equal by picking a low number of points from the functions, while two visually similar functions can be made to test unequal by picking a large number of points. Several other tests and procedures were tried, including correlation tests, linear regression tests, and tests involving the coefficients of polynomials fitted to the points of the functions. Results were the same in each case, with the only curves testing different being those almost completely opposite in form from one another. There were no clearly defined points for division into groups. The curves finally used for the main round of questioning were those depicted by the experts in the preliminary round which were visually different, along with some others added to give as complete a set of distinguishable curves as possible in the interval allowed.

This final set was then submitted to the experts for their evaluation. Detailed instructions were included, which explained exactly what was desired of the judges, and which tried to convey the concept of the self-appraised confidence estimates. It has been found that the degree of understanding by the judges of the procedure involved and of the information desired of them has a great effect on the degree of accord of their opinions.

While there is no attempt to force convergence of opinion (as in the DELPHI method) by trying to influence the votes of those judges with extreme opinions, it is felt that the comparison of his opinion with the opinions expressed by the other experts in the preliminary round will possibly either solidify his opinion as truly being the one he represented in the preliminary round, or will cause re-evaluation and a different vote with a higher personal confidence estimate. At the same time, by conducting the preliminary round, each judge is assured of seeing his opinion in the selection set of the main questionnaire, thus minimizing the error incurred when a judge must vote for a function which is not exactly the best in his opinion, simply because his opinion is not represented.

Analysis of Data and Decision

With receipt of the judges' responses, the period of analysis and decision begins. It is in this area that the method is quite unlike any other method yet developed.

There is a hazard associated with the use of averages of expert opinion without some try for consistency. If only one questionnaire is submitted to a large number of experts and the results are then averaged, there is a very good chance that any significance that might be present will be averaged out because of the problem of misunderstanding or semantics. The sensitivity of near-average values will be lost due to the variance associated with the estimates, which is increased when one or more rather extreme opinions are expressed. This particular problem can be alleviated to some extent by the use of some common method for deletion of extreme values. Admittedly this will introduce some biases, but past experience in this area shows it to be acceptable to the experts involved. In addition, it eliminates certain personal biases which may be introduced as specific points within the data collection. For example, in using non-confrontation schemes for collection of the data, there is still some "cancelling out" of the known positions of other persons on the expert panel. Obviously this could be taken into account if it were known to exist. This supports the elimination of extreme opinions, and from a sensitivity standpoint is clearly superior to the averaging of large groups.

To minimize the dangers inherent in simple averaging of the votes of the judges, a test has been incorporated in the new method that prevents any averaging unless the distribution of the votes is highly non-random. The probability associated with any specific pattern of votes can be calculated by evaluation of the multinomial distribution, when it is assumed that any member of the selection set is equally likely to be chosen by any judge.

The multinomial distribution [9] is associated with repeated trials of an event which can have more than two outcomes. In the general case, suppose that the event is repeated n times, and let the probabilities of the k possible outcomes be $p_1, p_2, \ldots, p_k$. Let x_1 be the number of times the outcome associated with p_1 occurs, x_2 the number of times the outcome associated with p_2 occurs, etc. Then the density function for the random variables $x_1, x_2, \ldots, x_{k-1}$ is given by

$$f(x_1, x_2, \ldots, x_{k-1}) = \frac{n!}{\prod_{i=1}^{k} x_i!} \prod_{i=1}^{k} p_i^{x_i} \qquad x_i = 0, 1, \ldots, n \qquad \sum_{i=1}^{k} x_i = n \quad .$$

For the specific case at hand, let n be the number of experts voting, let k be the number of functions in the selection set, and let $p_1=p_2= \ldots =p_k = 1/k$ be the probability that any specific function will be chosen when an expert casts his vote. Hence the above equation reduces in this case to

$$f(x_1, x_2, \ldots, x_{x-1}) = \frac{1}{\prod_{i=1}^{k} x_i!} \quad \frac{n!}{k^n} \quad .$$

From the requirements expressed in the formulation, namely that all the p_i's be equal, it can be seen that complete independence of the experts' votes is essential.

Of interest to this procedure is the probability that at most m of the functions will receive a vote, where m is the number of functions which do receive at least one vote. If this probability is extremely low (less than or equal to .05), it can be concluded that the distribution of the votes of the experts is non-random.

When the votes have been counted and recorded, this probability can be computed and the decision made to proceed in one of three directions. These directions are as follows.

If the probability is very low, a new function can be created which is a combination of the functions which received votes. This combination is a weighted average of the functions, with the product of the self-appraised confidence estimates and the reliability estimates being weights (w_i), and is given by

$$\overline{y}_e = \frac{\sum_{i=1}^{n} w_i \, y_{ei}}{\sum_{i=1}^{n} w_i}$$

where y_{ei} denotes the subjective estimate of Judge (i) of the value of the quantity under consideration. It should be pointed out that, in this application, the functions were combined by a weighted average of the ordinate values at each of eleven equally spaced abscissa points of the percent cost/percent time curves. The nature of the function in final form was to be monotonically increasing. A polynomial of degree three was fitted by a least squares approximation through the eleven computed ordinate values to produce the desired function, since such a polynomial was found to be most compatible with the nature of the final function.

If the computed probability is relative high (.30 or greater), it can only be concluded that the judges are in such a state of disagreement on the form of the function that any combination of their votes would fall prey to the dangers of averaging of large groups discussed previously, namely loss of significance due to a few extreme votes. This lack of accord may be due to differences in the experience bases of the experts, and if another attempt is made to predict this particular event by this method, the panel of experts should be chosen more carefully with regard to their backgrounds in the field.

If the probability is low but not low enough to be declared completely non-random (in this case in the range .06 to .29), logic dictates that some procedure be employed to eliminate extreme votes and reduce the number of functions which receive votes, thus reducing the probability of occurrence. To accomplish this purpose with a minimum of statistical involvement, it was decided that the votes for the function receiving the least number of votes should be eliminated. In case of a tie, the function (or functions) receiving both the least number of votes and the lowest weighting (sum of products of self-appraised confidence estimates and reliability estimates for the particular function) should be disregarded in further analysis. After this reduction in votes has taken place, the number of votes remaining should be compared to the original number of votes, and if this ratio becomes excessively low (this application used .70), the same general conclusions of differences in experience bases can be justified. If the ratio is still high, a new probability of occurrence can be calculated, again with the three possible outcomes. This procedure is repeated until the probability of occurrence falls to the acceptable range, or until the number of votes thrown out becomes excessive.

In the case where the probability does reach the acceptable range and the weighted means have been calculated for each of the eleven points, a variance estimate can be computed, given by

$$V(\bar{y}_e) = V(y_e) \frac{\sum_{i=1}^{n} w_i^2}{\left(\sum_{i=1}^{n} w_i\right)^2}$$

where

$$V(y_e) = \frac{\sum_{i=1}^{n} y_{ei}^2 - \frac{\left(\sum_{i=1}^{n} y_{ei}\right)^2}{n}}{n-1} .$$

Assuming normality of $\bar{y}_e$ about the true mean, approximate ninety percent confidence bands may be formed, given by

$$\bar{y}_e - 1.645 \sqrt{V(\bar{y}_e)} < \bar{y} < \bar{y}_e + 1.645 \sqrt{V(\bar{y}_e)} \qquad [10]$$

where $\bar{y}$ is the true mean. In this application, a polynomial was again fitted by least squares approximation, using the upper and lower confidence limits, to form continuous confidence bands.

Should either of the latter two outcomes of the probability test occur, namely probability of occurrence too high or too many votes eliminated, one possible course of action would be to rely on the DELPHI method of feedback to attain convergence.

In an experiment conducted at the Rand Corporation by Brown and Helmer, an attempt was made to improve upon the DELPHI method by revealing to each judge at the start of each round such information as the mode, median, and interquartile range of the subjestive values submitted by the judges in the last round. If any judge's opinion did not fall within the interquartile range, he was asked to state specifically his reasons for the deviation. His reasons were then anonymously made known to the other experts in the next round.

Several important results were cited by Brown and Helmer from this experiment:

1) Convergence occurred, in most cases, quite rapidly, with the interquartile range of the fourth round of questioning averaging only one-half that of the first round;

2) While convergence was quite noticeable, the opinions of the experts did not, in all cases, converge to the true answer;

3) In most cases results from data which were weighted with confidence estimates of the judges converged to a value much closer

to the true answer than the results of non-weighted data [11].

As in the Brown and Helmer experiment, an analysis which ends for either of the reasons given could be followed up with a second questionnaire, giving the median, mode, and interquartile range of the results of the main round of questioning, and asking for possible re-evaluation with these figures in mind.

An alternate procedure, in the event analysis cannot continue, would be to re-submit the selection set to those judges whose votes were eliminated or whose votes were not "in the ball park" before elimination occurred. They would be requested to restrict their vote to one of the functions which had received a sufficient number of votes to have remained under consideration at the time the number of votes deleted became excessive. In this case, realization of the fact that these judges are not actually voting for their true choice suggests that each of the confidence estimates of the judges in this group be automatically set to a minimum. In this way, the effect on the final function and its variance of a judge's vote in this group will be a minimum.

Conclusion

A computer program has been written in the FORTRAN IV language to facilitate automatic analysis of the data. In the event the analysis cannot be completed for either of the reasons given as the latter two outcomes of the probability test, the information is produced as output which is necessary to pursue either the Brown-Helmer approach or the approach of re-submission of the selection set for re-evaluation by the holders of extreme opinions. Also provided in the computer program is a means of testing the sensitivity of the results obtained by changing the vote, confidence estimate, reliability estimate, or any combination of the three, for any judge or judges. The input and utilization of the program are discussed in Appendix I.

There are admittedly some limitations on the use of this method. Foremost among these is the inability of the method to cope with a situation in which the data is not quantifiable, and thus not averageable.

While not completely consistent with regard to classical statistical theory, it is felt that this method utilizes enough statistics to produce reliable results. At the same time it retains enough simplicity of procedure to make it appealing to the statistical novice for whom it was intended, and by whom it was created.

APPENDIX I

UTILIZATION OF THE COMPUTER PROGRAM

Introduction

The main program for any given application of this method will differ from that of any other different application, simply because of the wide range of uses for the method. However, the subprograms which calculate the multinomial probabilities will remain the same for any application.

Basically, there are three types of data input to the main program provided for this application of the method. These are:

1) the curve set or selection set, which was the subject of the voting by the experts,

2) the votes, reliability estimates, and the confidence estimates for each of the judges, and

3) changes in the vote, reliability estimate, or confidence estimate of any number of judges, to provide a means for a sensitivity analysis.

The selection set is input as the ordinate values of the ten abscissa points 0.1, 0.2, . . . , 1.0. From the definition of the problem, namely the prediction of percent cost/percent time curves for future space projects, it is obvious that the ordinate value of the abscissa point 0.0 must also be 0.0 (no money can be spent before the start of the project), and the program automatically sets this ordinate value to 0.0.

The vote of each judge is input as a number which corresponds to the function of that judge's choice. The reliability estimate of each judge and also his confidence estimate are input as fractions with two decimal digits, such as 0.75.

The sensitivity analysis is accomplished by supplying input information on the change to be made, the number of the judge affected, and the new value of the variable to be changed. Any number of changes are allowed.

Input Formats

Type I Data - Selection Set Input. As has been said, the selection set is input in the form of ten ordinate values with the eleventh value automatically set to zero. The ten points of each curve are input, as floating point numbers, on two cards, five values per card, thus requiring 2k+1 cards for a selection set of k curves.

Card No.	Data	Card Cols.
1	The letters "CURVES"	1- 6
	k, the number of curves in the selection set, an integer value right justified	8- 9
2	Ordinate value of abscissa point 0.1 of the first curve in the selection set	7-12

	Ordinate value of abscissa point 0.2 of the first curve in the selection set	19-24
	Ordinate value of abscissa point 0.3 of the first curve in the selection set	31-36
	Ordinate value of abscissa point 0.4 of the first curve in the selection set	43-48
	Ordinate value of abscissa point 0.5 of the first curve in the selection set	55-60
3	Ordinate value of abscissa point 0.6 of the first curve in the selection set	7-12
	Ordinate value of abscissa point 0.7 of the first curve in the selection set	19-24
	Ordinate value of abscissa point 0.8 of the first curve in the selection set	31-36
	Ordinate value of abscissa point 0.9 of the first curve in the selection set	43-48
	Ordinate value of abscissa point 1.0 of the first curve in the selection set	55-60
4	Ordinate value of abscissa point 0.1 of the second curve in the selection set	7-12
	Ordinate value of abscissa point 0.2 of the second curve in the selection set	19-24
	⋮	
(2k)	Ordinate value of abscissa point 0.1 of the kth curve in the selection set	7-12
	⋮	

(2*k*+1)	. . . .	
	Ordinate value of abscissa point 1.0 of the *kth* curve in the selection set	55-60

Type II Data - Votes, Reliability Estimates, and Confidence Estimates. This type of input consists of a card for each participating judge. It is important to note that the order of the judges' votes as they are input to the computer correspond to the numbering of the judges, *e.g.*, the card for Judge (1) first, for Judge (2) second, . . . , Judge (n) last. This is to insure proper correspondence for any sensivity analysis which may be performed.

Data	Card Cols.
Number of the function in the selection set which received the vote of this judge, an integer value, right adjusted	1- 2
Confidence estimate of this judge, a floating point number	11-20
Reliability estimate for this judge, a floating point number	21-30

Type III Data - Sensitivity Analysis. There are three possible changes that can be made to test the sensitivity of the averaged function. These are a change in the vote of a judge, a change in the reliability estimate associated with a judge, and a change in the confidence estimate given by any judge. There is a unique card which must be input for each change.

	Data	Card Cols.
1)	The letters "CHANGE"	1- 6
	The letters "VOTE" to change the vote of a judge	8-11
	Number of the judge effected, an integer value, right justified	15-16
	Number of function to which this judge's vote is to be changed, an integer value, right justified	28-29
2)	The letters "CHANGE"	1- 6
	The letters "CONFID" to change the self-appraised confidence estimate of a judge	8-13
	Number of the judge effected, an integer value, right justified	15-16
	New value of the confidence estimate, a floating point number	20-29
3)	The letters "CHANGE"	1- 6
	The letters "RELIAB" if the reliability estimate of a judge is to be changed	8-13
	Number of the judge effected, an integer value, right justified	15-16
	New value of the reliability estimate, a floating point number	20-29

The initial input to the program must be the selection set, the Type I input. The last card of the selection set must be followed by a blank card, which in turn is followed by the first of the Type II inputs, the voting of the judges. A blank card must also be placed after the last voting card. Analysis will immediately begin. After completion of the analysis, there is a choice of further program direction. Another selection set may be input, followed by more voting cards. Another set of voting cards to apply to the current selection set may be processed. A sensitivity analysis may be performed on the current selection set and the most recent set of votes. After the completion of a sensitivity analysis, these same three directions are again available. After any sensitivity analysis, the votes, reliability estimates, and confidence estimates of all judges are returned to their original values, so that consecutive sensitivity analyses on the same votes are independent.

The only imperative conditions of input are that the inputting of a set of voting cards always follow the inputting of a new selection set, and that all of the three types of input always be followed by a blank card. The blank card signals to the program the termination of an input type.

Output

The output consists of only that information pertinent to the problem. The votes of the judges, along with any votes thrown out, are shown, and the probability of occurrence of this distribution is made known. After the analysis, the coefficients of the averaged function and the confidence bands are shown, along with the functions and confidence bands plotted by the off-line printer.

APPENDIX II

PROGRAM FUNCTIONS AND LISTINGS

Main Program

The purpose of the main program in this application is to perform all calculations other than multinomial probability calculations and polynomial fitting. The program reads all of the input data, determines if analysis is possible from the multinomial probability subprogram, and throws out extreme opinions of the judges in an attempt to lower this probability. If analysis is impossible, appropriate messages are produced, along with a record of the analysis up to the point of termination. If analysis continues, the mean and standard deviation of each of the eleven ordinate values are computed. Through the polynomial fitting subprogram, a cubic with a constant term of zero is fitted to the mean values and to points 1.645 standard deviations on either side of the mean values. Then by means of a curve plotting routine for the off-line printer, these cubic equations are plotted to produce smooth curves. Any sensitivity analysis required is also performed by the main program.

```
$IBFTC KATY
      INTEGER ORGNAL
      DIMENSION ORGNES(20), ORESJG(20)
      DIMENSION ORGNAL (15)
      DIMENSION CONEST(15),JUDGE(15)
      DIMENSION T(11)
      DIMENSION CELL(20), OBS(20,11),SUMX(11), SUMX2(11), IMAGE(1500),
     1  X(86), Y(86), NSCALE(5)
      DATA REDUCE /0.70/
      DATA (NSCALE(I),I=1,5)/1,0,2,0,2/
      DATA (T(I),I=1,11)/0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0/
      DATA CHANGE,CURVES/6HCHANGE,6HCURVES/
      DATA VOTE,CONFID/6HVOTE  ,6HCONFID/
      DATA CRTRIN/0.7/
      REAL JUDGES,JUDGE7
      IPROB = 0
 1001 READ (5,2002)  NUMBER
 2002 FORMAT (7X,I2)
 1000 FORMAT (I2,8X,7(F10.0))
      DO 666 I=1,NUMBER
      OBS(I,1) = 0.0
  666 READ (5,2000)  (OBS(I,J),J=2,11)
 2000 FORMAT (5(6X,F6.4))
      READ (5,2000)
   50 DO 100 I=1,NUMBER
      CONEST (I) = 0.0
  100 CELL(I) = 0.0
      JUDGES = 0.0
      I = 0
      IPROB = IPROB + 1
   10 READ (5,1000)  N,ESTMAT,ESTJDG
      IF (N .LT. 1)  GO TO 20
      IF (N .GT. NUMBER)  GO TO 10
      IF (ESTMAT .EQ. 0.0)  ESTMAT = 1.0
      IF (ESTJDG .EQ. 0.0)  ESTJDG = 1.0
```

```
      I = I+1
      ORGNES(I) = ESTMAT
      ORESJG(I) = ESTJDG
      JUDGE(I) = N
      ORGNAL(I) = N
      CONEST (N) = CONEST(N) +ESTMAT*ESTJDG
      JUDGES = JUDGES + 1.0
      CELL(N) = CELL(N) + 1.0
      GO TO 10
   20 CONTINUE
      JUDGCT = I
      ICODE = -1000
      NJUDG = JUDGES
      NOVOTE = 0
      JUDGE7 = JUDGES*REDUCE
 7600 CONTINUE
      WRITE (6,4500)  IPROB
 4500 FORMAT (1H1,55X,18HHISTORY - PROBLEM ,I2)
      WRITE (6,9876)
      WRITE (6,4510)  NUMBER
 4510 FORMAT (46X,36HNUMBER OF CURVES IN SELECTION SET = ,I2)
      WRITE (6,4520)  NJUDG
 4520 FORMAT (1H0,45X,19HNUMBER OF JUDGES = ,I2)
      WRITE (6,9876)
      DO 25 I=1,NUMBER
      IF (CELL(I) .EQ. 0.0)  NOVOTE = NOVOTE + 1
   25 CONTINUE
      Z = NUMBER
      CALL MULTI (Z,JUDGES,PROB,NOVOTE)
      IF (PROB .LT. CRTRIN)  GO TO 400
   30 IF (SUM(CELL,NUMBER) .LT. JUDGE7)  GO TO 500
      NOVOTE = 0
      DO 33 I=1,NUMBER
      IF (CELL(I) .EQ. 0.0)  NOVOTE = NOVOTE + 1
   33 CONTINUE
```

```
      JUDGES = SUM(CELL,NUMBER)
      CALL MULTI(Z,JUDGES,PROB,NOVOTE)
      IF (PROB .GE. 0.98)  GO TO 600
      ICODE = 1000
      VALUE = 1000.0
      DO 40  I=1,NUMBER
      IF ((CELL(I).GT.0.0) .AND. (CELL(I).LT.VALUE)) VALUE = CELL(I)
   40 CONTINUE
      VALUEC= 1000.00
      DO 41  I=1,NUMBER
      IF (.NOT.
     *   ( (CELL(I) .EQ. VALUE) .AND. (CONEST(I) .LT. VALUEC))) GOTO41
      VALUEC= CONEST(I)
   41 CONTINUE
      DO 42  I=1,NUMBER
      IF((CELL(I).NE.VALUE).OR.(CONEST(I).NE.VALUEC)) GO TO 42
      DO 39 K=1,JUDGCT
      IF (JUDGE(K) .EQ. I)  JUDGE(K) = 0
   39 CONTINUE
      CELL(I) = 0.0
      CONEST(I) = 0.0
   42 CONTINUE
      JUDGES = SUM(CELL,NUMBER)
      GO TO 30
  600 CONTINUE
      IF (ICODE) 700,601,800
  700 WRITE (6,4530)
 4530 FORMAT (59X,14H     VOTES         //)
      DO 701  I=1,JUDGCT
  701 WRITE (6,4540)  I, JUDGE(I)
 4540 FORMAT (59X,6HJUDGE ,I2,3H   ,I2)
  610 WRITE (6,9876)
      GOAHED = 1.0 - PROB
      WRITE (6,4550)  GOAHED
 4550 FORMAT (48X,28HPROBABILITY OF OCCURRENCE = ,F8.5)
```

```
      WRITE (6,4590)
 4590 FORMAT (1H0,56X,18HANALYSIS CONTINUES    )
      GO TO 601
  800 WRITE (6,4560)
 4560 FORMAT (51X,8HORIGINAL,14X,7HREVISED/
     *         51X,8H VOTES   ,14X,7H VOTES //)
      DO 810 I=1,JUDGCT
  810 WRITE (6,4570)  ORGNAL(I),I,JUDGE(I)
 4570 FORMAT (53X,I2,6X,6HJUDGE ,I2,6X,I2)
      GO TO 610
  601 DO 150 I=1,86
  150 X(I) = FLOAT(I)/86.0
      WRITE (6,9876)
      WRITE (6,3010)
 3010 FORMAT (50X,35HCOEFFICIENTS OF FITTED POLYNOMIALS         )
      WRITE (6,3020)
 3020 FORMAT (1H0,49X,4HX**3,15X,4HX**2,15X,4H  X ,15X,4H  C         )
 3000 FORMAT (1H1)
      DO 604 I=1,NUMBER
  604 CELL(I) = CONEST(I)
      JUDGES = SUM(CONEST,NUMBER)
      DO 612 I=1,11
      SUMX(I) = 0.0
  612 SUMX2(I) = 0.0
      DO 620 I=1,NUMBER
      IF (CELL(I) .LT. .05)  GO TO 620
      DO 630 J=1,11
      SUMX(J) = SUMX(J) + OBS(I,J) * CELL(I)
  630 SUMX2(J) = SUMX2(J) + CELL(I)*OBS(I,J)**2
  620 CONTINUE
      DO 640 I=1,11
      SUMX2(I) = SQRT((SUMX2(I)-(SUMX(I)**2/JUDGES))/(JUDGES-1.0))
     1  *1.645
  640 SUMX(I) = SUMX(I)/JUDGES
      CALL CUBIC (A,B,C,D,T,SUMX,11,1)
```

```
C     THE LAST ARGUMENT OF THE CALLING SEQUENCE OF -CUBIC- DETERMINES
C     IF A CONSTANT IS DESIRED...0=CONSTANT...1=NO CONSTANT
      WRITE (6,3030)  A,B,C,D
 3030 FORMAT (1H0,14X,17HAVERAGED FUNCTION,15X,4(E12.5,7X))
      DO 650 I=1,86
  650 Y(I) = X(I)*(X(I)*(A*X(I)+B)+C) +D
      CALL PLOT 1 (NSCALE,5,10,5,17)
      CALL PLOT 2 (IMAGE,1.0,0.0,1.0,0.0)
      CALL PLOT 3 (1H*,X,Y,86)
      DO 655 I=1,11
  655 SUMX(I) = SUMX(I) + SUMX2(I)
      CALL CUBIC (A,B,C,D,T,SUMX,11,1)
      WRITE (6,3040)  A,B,C,D
 3040 FORMAT (1H0,14X,22HUPPER CONFIDENCE LIMIT,10X,4(E12.5,7X))
 3050 FORMAT (1H0,14X,22HLOWER CONFIDENCE LIMIT,10X,4(E12.5,7X))
      DO 660 I=1,86
  660 Y(I) = X(I)*(X(I)*(A*X(I)+B)+C) +D
      CALL PLOT 3 (1H.,X,Y,86)
      DO 665 I=1,11
  665 SUMX(I) = SUMX(I) - 2.0*SUMX2(I)
      CALL CUBIC (A,B,C,D,T,SUMX,11,1)
      WRITE (6,3050)  A,B,C,D
      DO 670 I=1,86
  670 Y(I) = X(I)*(X(I)*(A*X(I)+B)+C) +D
      CALL PLOT 3 (1H.,X,Y,86)
      WRITE (6,3000)
      CALL PLOT 4 (37,37H              P E R C E N T   C O S T        )
      WRITE (6,9397)
 9397 FORMAT (1H0,43X,24HP E R C E N T   T I M E              )
      GO TO 7000
  500 WRITE (6,4560)
      DO 510 I=1,JUDGCT
  510 WRITE (6,4570)  ORGNAL(I),I,JUDGE(I)
      NJUDG = JUDGES
      WRITE (6,9876)
```

```
      WRITE (6,9877)  NJUDG
 9877 FORMAT (1H0,50X,28HNUMBER OF JUDGES REDUCED TO     ,I2)
      WRITE (6,4580)
      GO TO 7000
  400 WRITE (6,4530)
      DO 401 I=1,JUDGCT
  401 WRITE (6,4560)  I,ORGNAL(I)
  900 GOAHED = 1.0 - PROB
      WRITE (6,9876)
      WRITE (6,4550)  GOAHED
      WRITE (6,4580)
 4580 FORMAT (1H0,56X,18HANALYSIS ABANDONED    )
      GO TO 7000
 9876 FORMAT (1H0/1H0)
 6789 FORMAT (10X,11(3X,F8.4))
 7000 READ (5,7001)  CODE
 7001 FORMAT (A6)
      IF (CODE .EQ. CURVES)  GO TO 9000
      IF (CODE .EQ. CHANGE ) GO TO 7002
      BACKSPACE 5
      GO TO 50
 7002 CONTINUE
      DO 7010 I=1,NUMBER
      CELL(I) = 0.0
 7010 CONEST(I) = 0.0
      DO 7020 I=1,JUDGCT
      JUDGE(I) = ORGNAL(I)
      N = ORGNAL(I)
      CELL(N) = CELL(N) + 1.0
 7020 CONEST(N) = CONEST(N) + ORGNES(I) * ORESJG(I)
 7080 BACKSPACE 5
      READ (5,7050)  CODE,I,WHAT
 7050 FORMAT (7X,A6,1X,I2,3X,F10.0)
      N = ORGNAL (I)
      IF (CODE .EQ. VOTE)  GO TO 7060
```

```
      IF (CODE .EQ. CONFID)  GO TO 7070
      CONEST(N) = CONEST(N) - ORGNES(I) * ORESJG (I)
      CONEST(N) = CONEST(N) + ORGNES(I) * WHAT
      GO TO 7500
 7060 CELL(N) = CELL(N) - 1.0
      K = WHAT
      JUDGE(I) = K
      CELL(K) = CELL(K) + 1.0
      CONEST(N) = CONEST(N) - ORGNES(I) * ORESJG(I)
      CONEST (K) = CONEST (K) + ORGNES(I) * ORESJG (I)
      GO TO 7500
 7070 CONEST(N) = CONEST(N) - ORGNES(I) * ORESJG (I)
      CONEST(N) = CONEST(N) + WHAT*ORESJG(I)
 7500 READ (5,7001)  CODE
      IF(CODE .EQ. CHANGE)  GO TO 7080
      JUDGES = SUM (CELL,NUMBER)
      GO TO 7600
 9000 BACKSPACE 5
      GO TO 1001
      END
```

Subroutine CUBIC

CUBIC is a subroutine which fits a third degree polynomial to a set of data points by means of a simple least squares fit. The calling sequence is

CALL CUBIC (A, B, C, D, X, Y, NUMBER, IFLAG)

where A, B, C, D are coefficients of the fitted polynomial such that

$$y_i = Ax_i^3 + Bx_i^2 + Cx_i + D$$

X = array of abscissa values

Y = array of ordinate values

NUMBER = number of X-Y points in fit

$$\text{IFLAG} = \begin{cases} 0 \text{ if D is to be calculated} \\ 1 \text{ if D is to be forced to zero .} \end{cases}$$

```
$IBFTC FITTER  DECK
      SUBROUTINE CUBIC(A,B,F,D,X,Y,N,M)
      DIMENSION PROD(4,25), XBAR(25,4), T(4), C(4,4), XTRANS(4,25)
      DIMENSION IPIV(4)
      DIMENSION X(N), Y(N)
      DATA NOLD /10/
      L = 4-M
      IF (NOLD .EQ. N)  GO TO 20
      NOLD = N
   10 DO 200 I=1,N
      DO 200 J=1,L
      XBAR(I,J) = X(I)**(M+J-1)
  200 XTRANS(J,I) = XBAR(I,J)
      DO 300 I=1,L
      DO 300 J=1,L
      C(I,J) = 0.0
      DO 300 K=1,N
  300 C(I,J) = C(I,J) + XTRANS(I,K)*XBAR(K,J)
      A = TAMINV(C,IPIV,L,4,1.0E-10)
      DO 500 I=1,L
      DO 500 J=1,N
      PROD(I,J) = 0.0
      DO 500 K=1,L
  500 PROD(I,J) = PROD(I,J) + C(I,K) * XTRANS(K,J)
   20 DO 700 I=1,L
      T(I) = 0.0
      DO 700 J=1,N
  700 T(I) = T(I) + PROD(I,J) * Y(J)
      A = T(L)
      B = T(L-1)
      F = T(L-2)
      D = T(L-3)
      IF (L .EQ. 3)  D = 0.0
      RETURN
      END
```

Subroutine MULTI

Subroutine MULTI is the main program for the calculation of multinomial probabilities. To accomplish this, it communicates with subroutines SUM, N1OR2, TERM, INCREM, and FACTOR.

The probability of exactly q of the choices in the selection set not receiving a vote is given by

$$P_q = P\text{ (exactly } \underline{q} \text{ with no vote)} = \sum_{i=1}^{t} C_j f(x_{j1}, x_{j2}, \ldots, x_{jn})$$

where

$\underline{t}$ = number of distinct sets of x_i such that $\underline{q}$ of the x's are zero

C_j = number of permutations of $\underline{n}$ x's taken $\underline{n}$ at a time (some of the x's can be the same) for the given set of x_i, which is given by

$$\frac{N!}{r_o!r_1! \ldots r_k!} \qquad r_o = q \qquad 0 \leq m \leq k$$

$$r_m \begin{cases} = S \text{ if the integer } \underline{m} \text{ appears S times in the } \underline{jth} \text{ set of x's} \\ = 0 \text{ if the integer } \underline{m} \text{ does not appear in the } \underline{jth} \text{ set of x's} \end{cases}$$

and x_i = number of votes received by the ith function in the selection set.

Thus, the probability that we desire, namely that q or more of the functions will not receive a vote, is given by

$$P_q = 1.0 - \sum_{i=0}^{q-1} p_i$$

To accomplish the above calculations, the program builds tables of all of the possible ways that _n_ votes can be distributed among _k_ functions with _q_ of the functions not receiving a vote.

An example of the table built is given below for the case of twelve votes being distributed among ten functions with five of the possible functions not receiving a vote. As can be seen, there are thirteen possible distributions, thus for this case _t_ = 13.

8	1	1	1	1	0	0	0	0	0
7	2	1	1	1	0	0	0	0	0
6	3	1	1	1	0	0	0	0	0
5	4	1	1	1	0	0	0	0	0
6	2	2	1	1	0	0	0	0	0
5	3	2	1	1	0	0	0	0	0
4	4	2	1	1	0	0	0	0	0
5	2	2	2	1	0	0	0	0	0
4	3	2	2	1	0	0	0	0	0
4	2	2	2	2	0	0	0	0	0
3	3	2	2	2	0	0	0	0	0
4	3	3	1	1	0	0	0	0	0
3	3	3	2	1	0	0	0	0	0

Table of Possible Distributions
for _n_ = 12, _k_ = 10, _q_ = 5

The calling sequence for subroutine MULTI is

CALL MULTI (CURVES, EXPERT, PROB, NOVOTE)

where CURVES is the number of curves in the selection set

EXPERT is the number of judges participating

PROB is returned equal to the probability of occurrence for this pattern of votes

NOVOTE is the number of possible selections which did not receive a vote.

```
$IBFTC SUB1
      SUBROUTINE MULTI (CURVES,JUDGES,TORAL,NOVOTE)
      REAL JUDGES
      INTEGER ZEROS
      INTEGER ROW
      DOUBLE PRECISION FACTOR,POWER
      DOUBLE PRECISION TERM
      DOUBLE PRECISION FINAL,ANSWER,TOTAL
      DIMENSION          X(20), TABLE(100,20)
      NCURV = CURVES
      NJUDG = JUDGES
      TOTAL = 0.0
      FINAL = (FACTOR(NJUDG))/CURVES**JUDGES
      K = CURVES - JUDGES + 1.0
      IF (K .LE. 0)  K=1
      L = CURVES
      DO 1000  INDEX = K,L
      ITRIG = 0
      ZEROS = INDEX - 1
      N = CURVES - FLOAT(ZEROS)
      IF ( (N.EQ.2) .OR. (N.EQ.1) ) GO TO 80
      DO 20 J=1,N
   20 X(J) = 1.0
      ROW = 0
      GO TO 35
   30 CALL INCREM (X,N)
   35 IF ( (SUM(X,N) .GT. JUDGES) .AND. (ITRIG .GT. 500) ) GO TO 60
      IF (SUM(X,N) . GT. JUDGES) GO TO 61
      X(1) = JUDGES - SUM(X,N) + X(1)
      GO TO 55
   50 X(1) = X(1) - 1.0
      X(2) = X(2) + 1.0
      IF (X(1) .LT. X(2)) GO TO 30
   55 ROW = ROW + 1
      DO 40 NN=1,N
```

```
   40 TABLE(ROW,NN) = X(NN)
      GO TO 50
   60 ANSWER = 0.0
      DO 65 J=1,ROW
   65 ANSWER = ANSWER + TERM(TABLE,J,N)
      ANSWER = ANSWER * FACTOR(NCURV)/(FACTOR(NCURV-ZEROS)*FACTOR(
     1  ZEROS))*FINAL    *FACTOR(N)
      TOTAL = TOTAL + ANSWER
      TORAL = TOTAL
      IF (NOVOTE .LE. ZEROS)  RETURN
 1000 CONTINUE
   80 CALL N1OR2 (TABLE,N,ROW,JUDGES)
      GO TO 60
   61 ITRIG = 1000
      DO 85 I=1,N
   85 X(I) = X(3)
      GO TO 30
      END
```

Subroutine TERM

From the example table of possible vote patterns, the probability of exactly five possible selections not receiving a vote from twelve judges, who consider a total of ten selections, can be calculated. This calculation is performed by the function subprogram TERM. The program takes each row of the possibility table and performs the calculation

$$P = \frac{1}{r_o!r_1! \ldots r_k!} \cdot \frac{1}{x_1!x_2! \ldots x_k!}$$

where r_i and x_i have the same meaning as in the discussion of subroutine MULTI. TERM uses double precision arithmetic because of the round-off which occurs in an operation of the magnitude $\frac{1}{15!} \cdot \frac{1}{15!}$.

The calling sequence for TERM is

P = TERM (TABLE, IROW, J)

where TABLE is the array of possible vote patterns

IROW is the row in TABLE under consideration

J is the number of elements in row IROW of TABLE .

```
$IBFTC SUB2
      DOUBLE PRECISION FUNCTION TERM (TABLE,ROW,N)
      DOUBLE PRECISION DENOM,FIRST
      DOUBLE PRECISION FACTOR,POWER
      DIMENSION TABLE(100,20), SAVE(20)
      INTEGER ROW
      VALUE = TABLE(ROW,1)
      DO 30 I=1,N
   30 SAVE(I) = 1.0
      I = 1
      DO 10 K=2,N
      IF (TABLE(ROW,K) .EQ. VALUE) GO TO 20
      I = I+1
      VALUE = TABLE(ROW,K)
   10 CONTINUE
      GO TO 60
   20 SAVE(I) = SAVE(I) + 1.0
      GO TO 10
   60 DENOM = 1.0
      DO 40 I=1,N
      L = SAVE(I)
      IF (L .EQ. 1)     GO TO 40
      DENOM = DENOM * FACTOR(L)
   40 CONTINUE
      FIRST = 1.0/DENOM
      DENOM = 1.0
      DO 50 I = 1,N
      L = TABLE(ROW,I)
      IF (L .EQ. 1)  GO TO 50
      DENOM = DENOM*FACTOR(L)
   50 CONTINUE
      TERM = FIRST/DENOM
      RETURN
      END
```

Subroutine INCREM

The table of possible patterns already discussed is built upon the idea that, for any row in the table, $x_2 \geq x_3 \geq x_4 \geq \ldots \geq x_n$, and $x_1 = n - \sum_{i=2}^{n} x_i$. In the example table given, the row

4 5 1 1 1 0 0 0 0 0

would have been repetitious of the row

5 4 1 1 1 0 0 0 0 0

and thus could not be used. Subroutine INCREM decides what combination of numbers should be entered in the next row of the table, and makes this information available to the main program. In this case the new row was

6 2 2 1 1 0 0 0 0 0 .

The calling sequence of subroutine INCREM is

CALL INCREM (ROW, J)

where ROW is the vector which has been determined to be repetitious of another vector

J is the number of elements in vector ROW.

```
$IBFTC SUB3
      SUBROUTINE INCREM(X,N)
      DIMENSION X(20)
      L = N+4
      DO 10 I=4,N
      K = L-I
      IF (X(K) .LT. X(K-1)) GO TO 20
   10 CONTINUE
      DO 30 I=4,N
   30 X(I) = 1.0
      X(3) = X(3) + 1.0
   40 X(1) = X(3)
      X(2) = X(3)
      RETURN
   20 X(K) = X(K) + 1.0
      GO TO 40
      END
```

Subroutine SUM

The function subprogram SUM performs the operation

$$SUM = \sum_{i=1}^{j} Z_i \; .$$

In the calculation of multinomial probabilities, SUM is used to determine if any row in the table of possible vote patterns contains more votes than the number of judges participating.

The calling sequence of SUM is given by

TOTAL = SUM (Z, J)

where Z is the row vector whose elements are to be summed, and J is the upper limit of the summation.

```
$IBFTC SUB4
      FUNCTION SUM(X,N)
      DIMENSION X(20)
      SUM = 0.0
      DO 10 I=1,N
   10 SUM = SUM + X(I)
      RETURN
      END
```

Subroutine N1OR2

The purpose of subroutine N1OR2 is to produce a table of possible vote patterns when the votes are concentrated on either one or two of the possible choices. This subroutine is required because of the inability of subroutine INCREM to handle the situation when _q_ is one or two.

The calling sequence of subroutine N1OR2 is

CALL N1OR2 (TABLE, J, IROW, VOTES)

where TABLE is the array of possible vote patterns

J is the number of selections which received a vote (1 or 2)

IROW is returned to the main program equal to the number of rows of TABLE filled in by subroutine N1OR2

VOTES is the number of votes cast .

```
$IBFTC SUB5
      SUBROUTINE N1OR2 (TABLE,N,ROW,JUDGES)
      DIMENSION TABLE (100,20)
      INTEGER ROW
      REAL JUDGES
      GO TO (1,2),N
    1 TABLE(1,1) = JUDGES
      ROW = 1
      RETURN
    2 ROW = 0
      X = 1.0
      Y = JUDGES - 1.0
   20 IF (Y .LT. X) GO TO 30
      ROW = ROW+1
      TABLE(ROW,1) = Y
      TABLE(ROW,2) = X
      X = X+1.0
      Y = Y-1.0
      GO TO 20
   30 RETURN
      END
```

Subroutine FACTOR

Subroutine FACTOR is a double precision function subprogram which answers

$$VALUE = FACTOR (I)$$

by setting VALUE equal to I! . This program does a table lookup of values of I! from I=0 to I=15 from a table of double precision constants.

```
$IBMAP FACTOS   NOLIST,NOREF,DECK
FACTOR SAVE     1,4
       CLA*     3,4
       ADD      =1
       PAC      ,1
       XEC      START,1
       RETURN   FACTOR
START  NOP
       DLD      =1.EE0
       DLD      =1.EE0
       DLD      =2.EE0
       DLD      =6.EE0
       DLD      =24.EE0
       DLD      =120.EE0
       DLD      =720.EE0
       DLD      =5040.EE0
       DLD      =40320.EE0
       DLD      =362880.EE0
       DLD      =3628800.EE0
       DLD      =39916800.EE0
       DLD      =479001600.EE0
       DLD      =6227020800.EE0
       DLD      =87178291200.EE0
       DLD      =1307674368000.EE0
       EVEN
       END
```

APPENDIX III

A SAMPLE PROGRAM

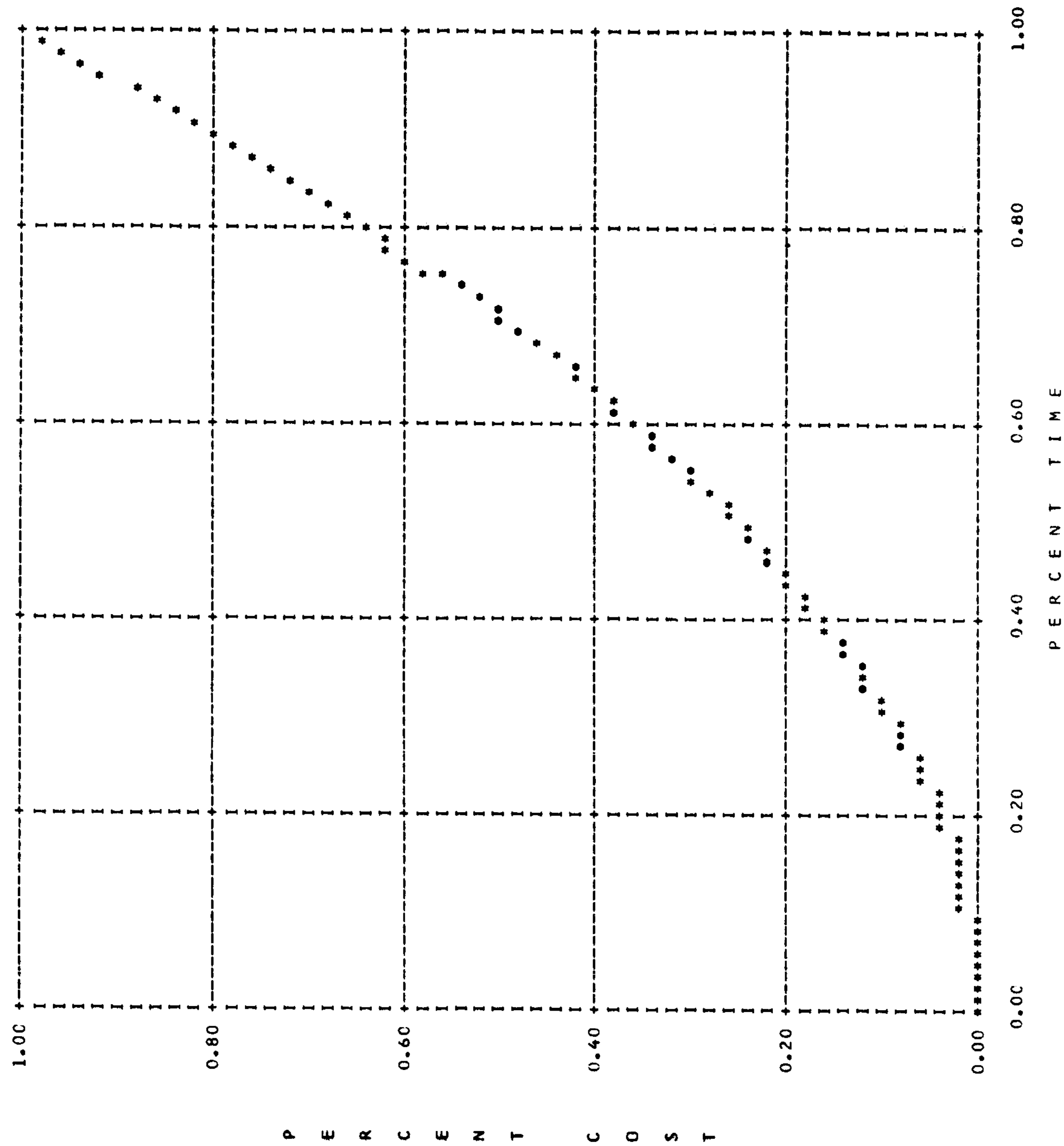
The following pages are offered as an example of the capabilities and incapabilities of the program. Shown is a listing of a set of sample data, the functions represented by the ordinate values of the selection set input, and the results. After several analyses have been performed using this selection set, a new selection set is input and followed by further analyses.

LISTING OF INPUT DATA FOR SAMPLE PROBLEM

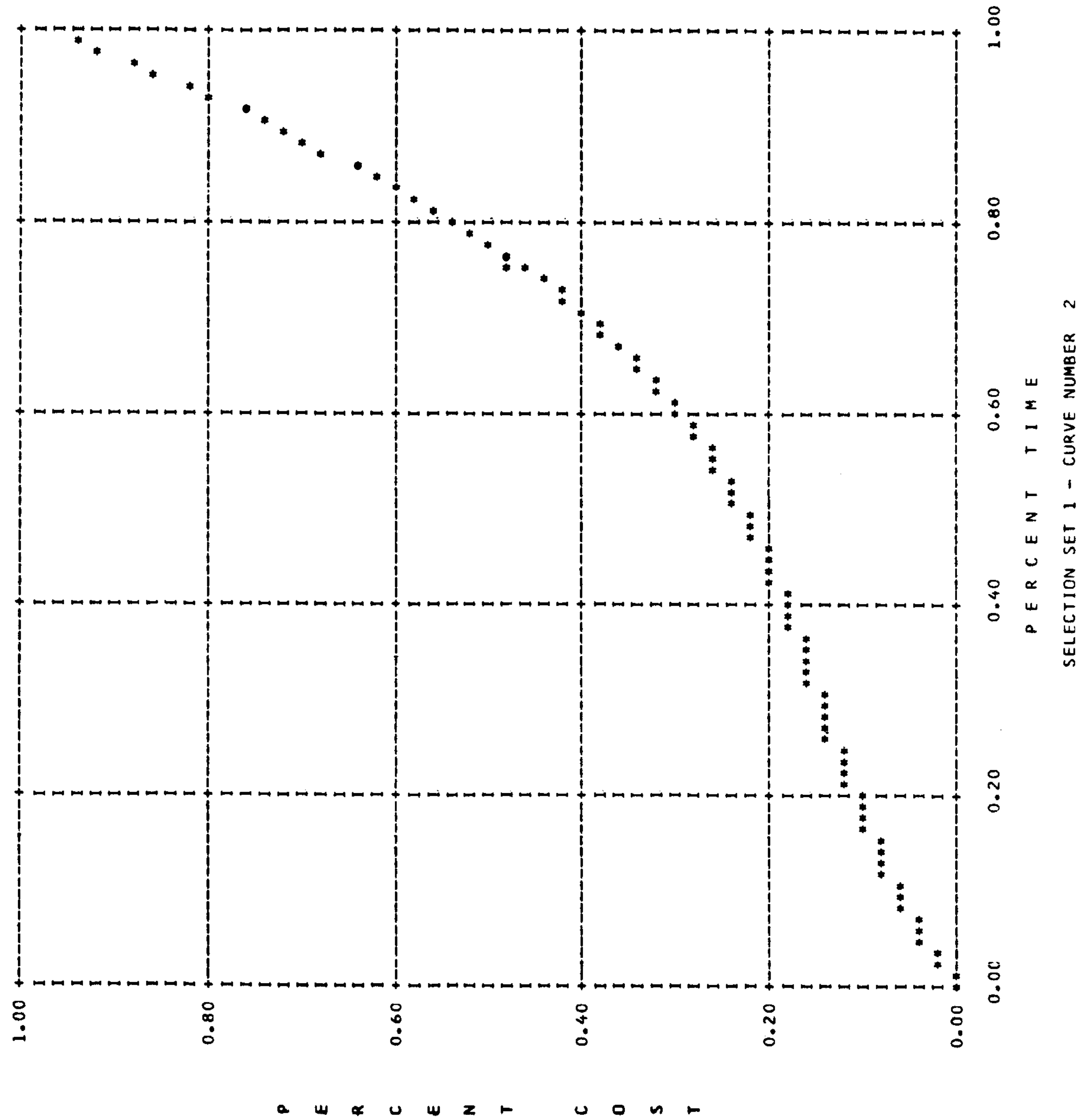
```
$DATA
CURVES 09
       0.0100      0.0400      0.0900      0.1600      0.2500
       0.3600      0.4900      0.6400      0.8100      1.0000
       0.0450      0.0850      0.1400      0.1950      0.2500
       0.3150      0.4000      0.5100      0.7000      1.0000
       0.1000      0.2000      0.3000      0.4000      0.5000
       0.6000      0.7000      0.8000      0.9000      1.0000
       0.0050      0.0100      0.0150      0.0250      0.0350
       0.0460      0.0850      0.2600      0.5800      1.0000
       0.0200      0.0400      0.0600      0.0800      0.1050
       0.1400      0.1800      0.2600      0.4000      1.0000
       0.2500      0.4400      0.5800      0.7000      0.7750
       0.8400      0.8900      0.9350      0.9700      1.0000
       0.2200      0.3650      0.4600      0.5250      0.5600
       0.5900      0.6250      0.6750      0.7600      1.0000
       0.0900      0.1700      0.2300      0.2800      0.3350
       0.3650      0.4000      0.4600      0.5750      1.0000
       0.0450      0.1000      0.1600      0.2490      0.3800
       0.6600      0.8350      0.9300      0.9750      1.0000
        (BLANK CARD)
01          0.75       0.10
03          0.50       0.20
05          0.75       0.30
07          1.00       0.40
01          0.25       0.50
03          0.25       0.60
01          0.40       0.70
01          0.75       0.80
01          0.95       0.90
03          0.50       1.00
        (BLANK CARD)
```

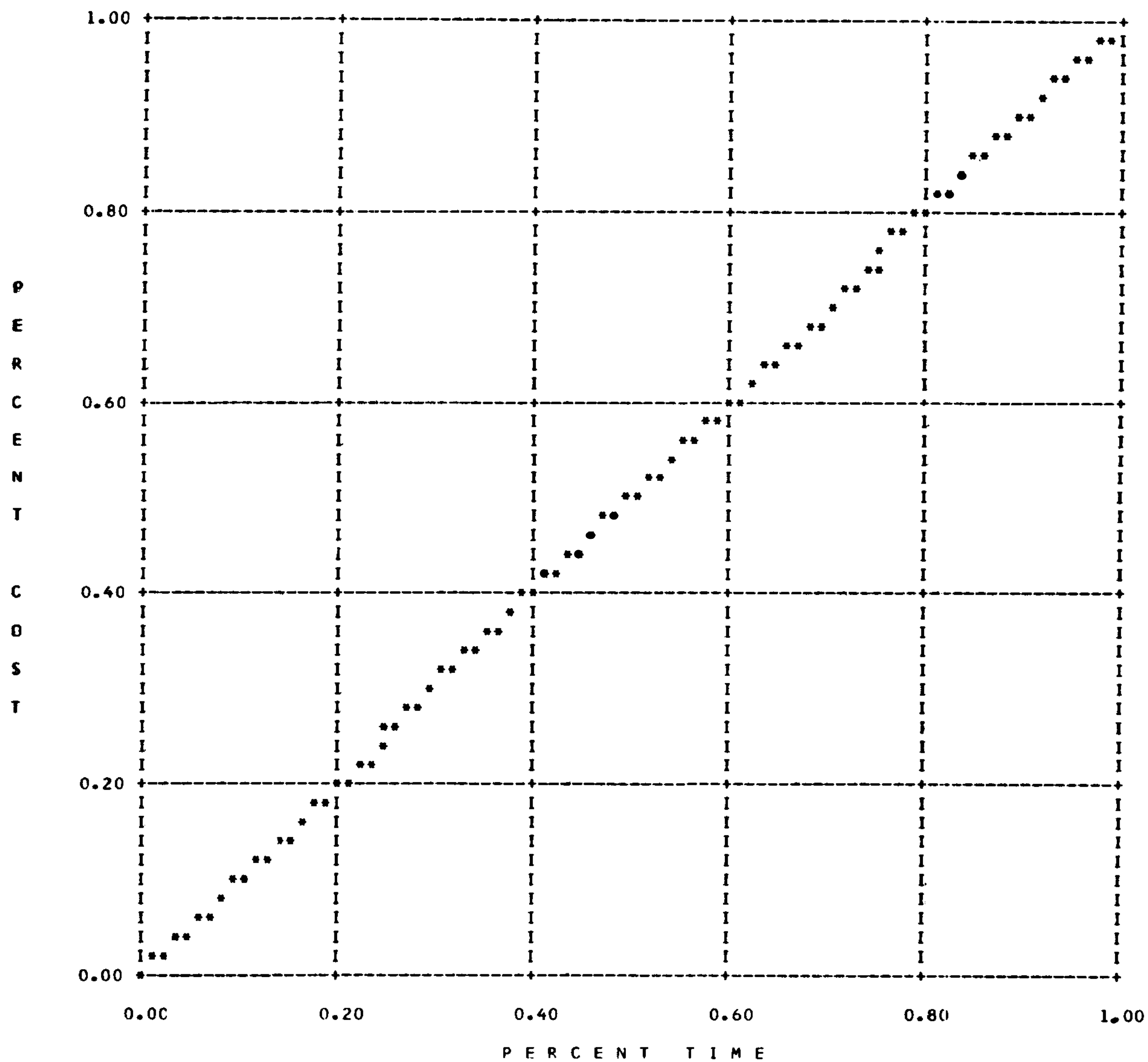
```
CHANGE VOTE   02             7
CHANGE VOTE   06             7
CHANGE CONFID 04    0.50
       (BLANK CARD)
CHANGE VOTE   02             5
CHANGE VOTE   06             5
CHANGE RELIAB 10    0.70
CHANGE CONFID 04    0.50
       (BLANK CARD)
09          0.25      0.25
09          0.50      0.75
04          0.45      0.50
06          0.50      0.60
08          0.80      0.90
02          0.75      0.75
02          0.80      0.50
08          0.75      0.80
09          0.65      0.75
06          0.75      0.65
04          0.40      0.50
01          0.30      0.25
       (BLANK CARD)
CHANGE VOTE   01             1
       (BLANK CARD)
CURVES 07
       0.0450      0.1000      0.1600      0.2490      0.3800
       0.6600      0.8350      0.9300      0.9750      1.0000
       0.0900      0.1700      0.2300      0.2800      0.3350
       0.3650      0.4000      0.4600      0.5750      1.0000
       0.2200      0.3650      0.4600      0.5250      0.5600
       0.5900      0.6250      0.6750      0.7600      1.0000
       0.2500      0.4400      0.5800      0.7000      0.7750
       0.8400      0.8900      0.9350      0.9700      1.0000
       0.1000      0.2000      0.3000      0.4000      0.5000
       0.6000      0.7000      0.8000      0.9000      1.0000
```

```
      0.0450      0.0850      0.1400      0.1950      0.2500
      0.3150      0.4000      0.5100      0.7000      1.0000
      0.0100      0.0400      0.0900      0.1600      0.2500
      0.3600      0.4900      0.6400      0.8100      1.0000
       (BLANK CARD)
02         0.65        0.70
03         0.95        0.50
07         0.75        0.35
07         0.50        0.65
07         0.50        0.75
02         0.60        0.95
03         0.75        0.80
07         0.70        0.60
05         0.40        0.50
       (BLANK CARD)
```



SELECTION SET 1 - CURVE NUMBER 1


1.00
0.80
0.60
0.40
0.20
0.00
0.00
0.20
0.40
0.60
0.80
1.00
P E R C E N T C O S T
P E R C E N T T I M E
SELECTION SET 1 - CURVE NUMBER 2



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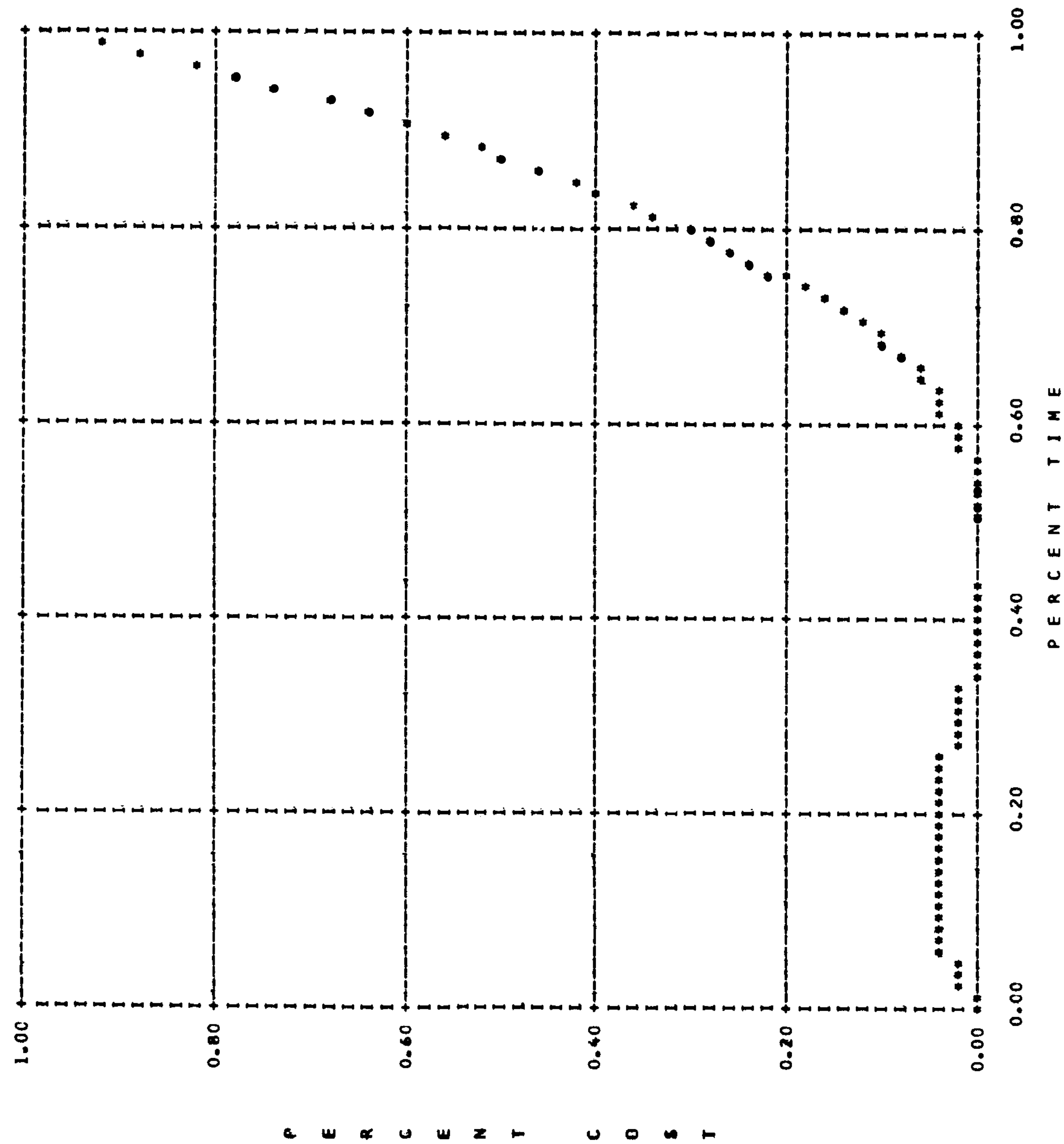
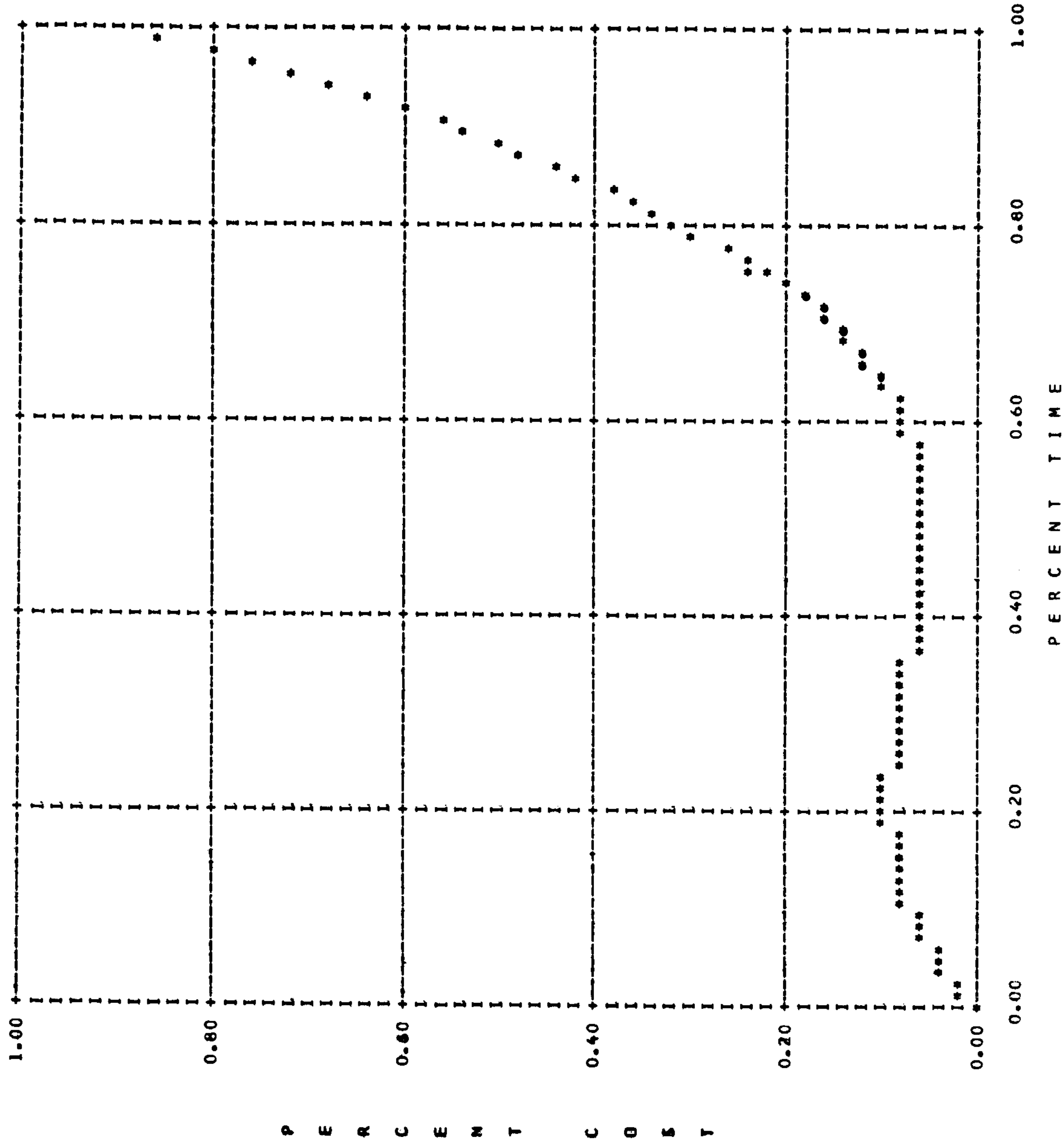


SELECTION SET 1 - CURVE NUMBER 4



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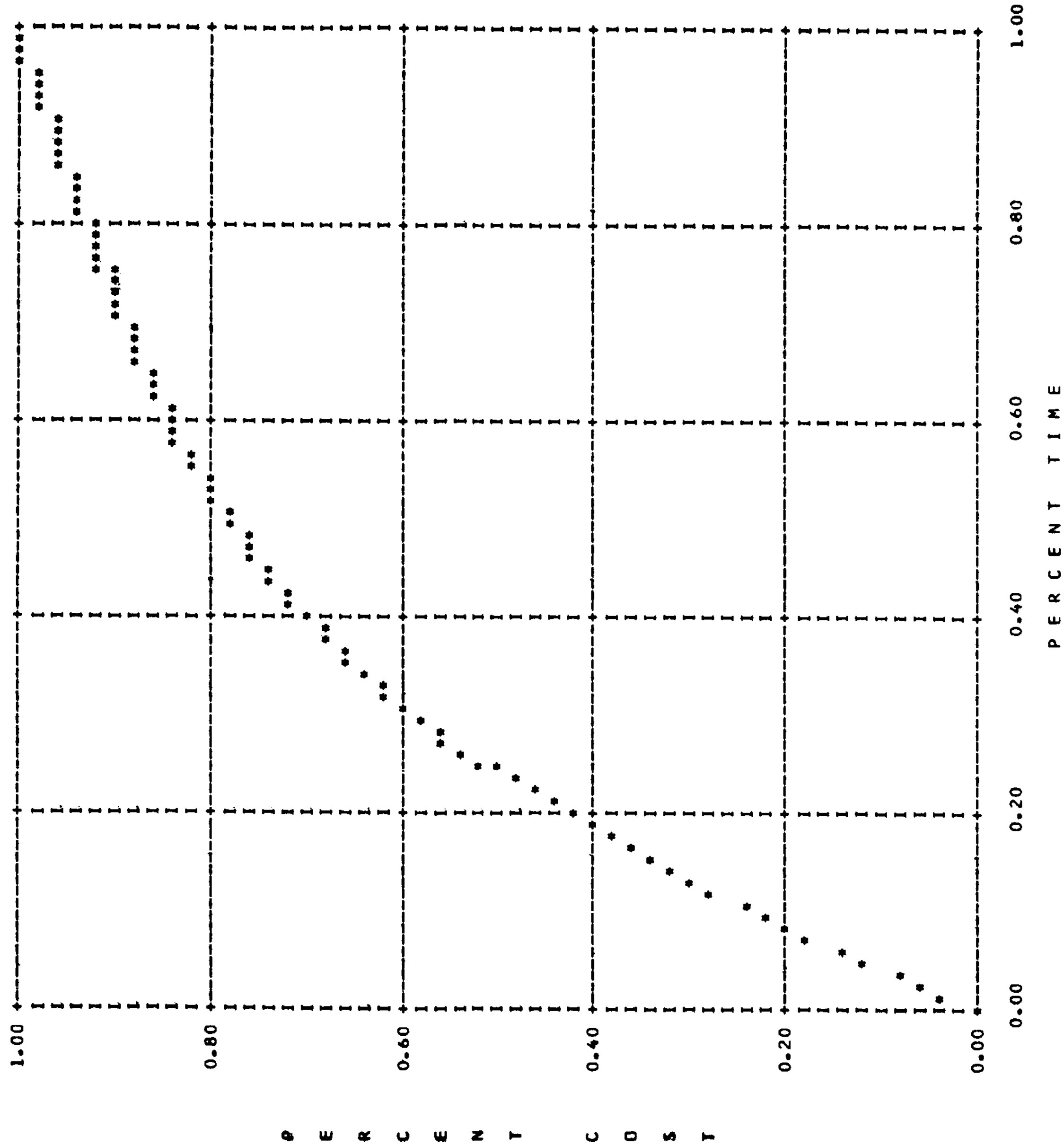

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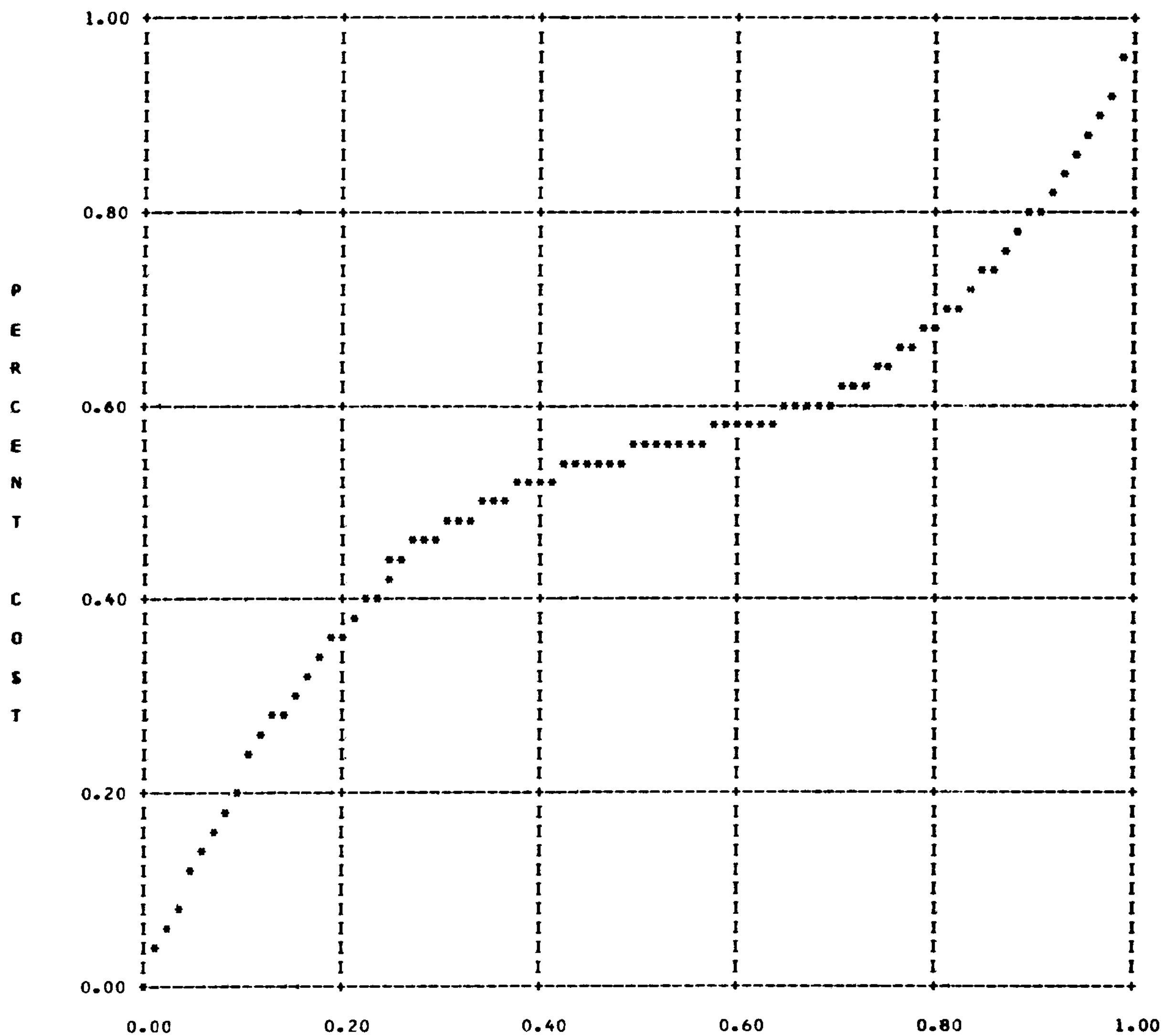
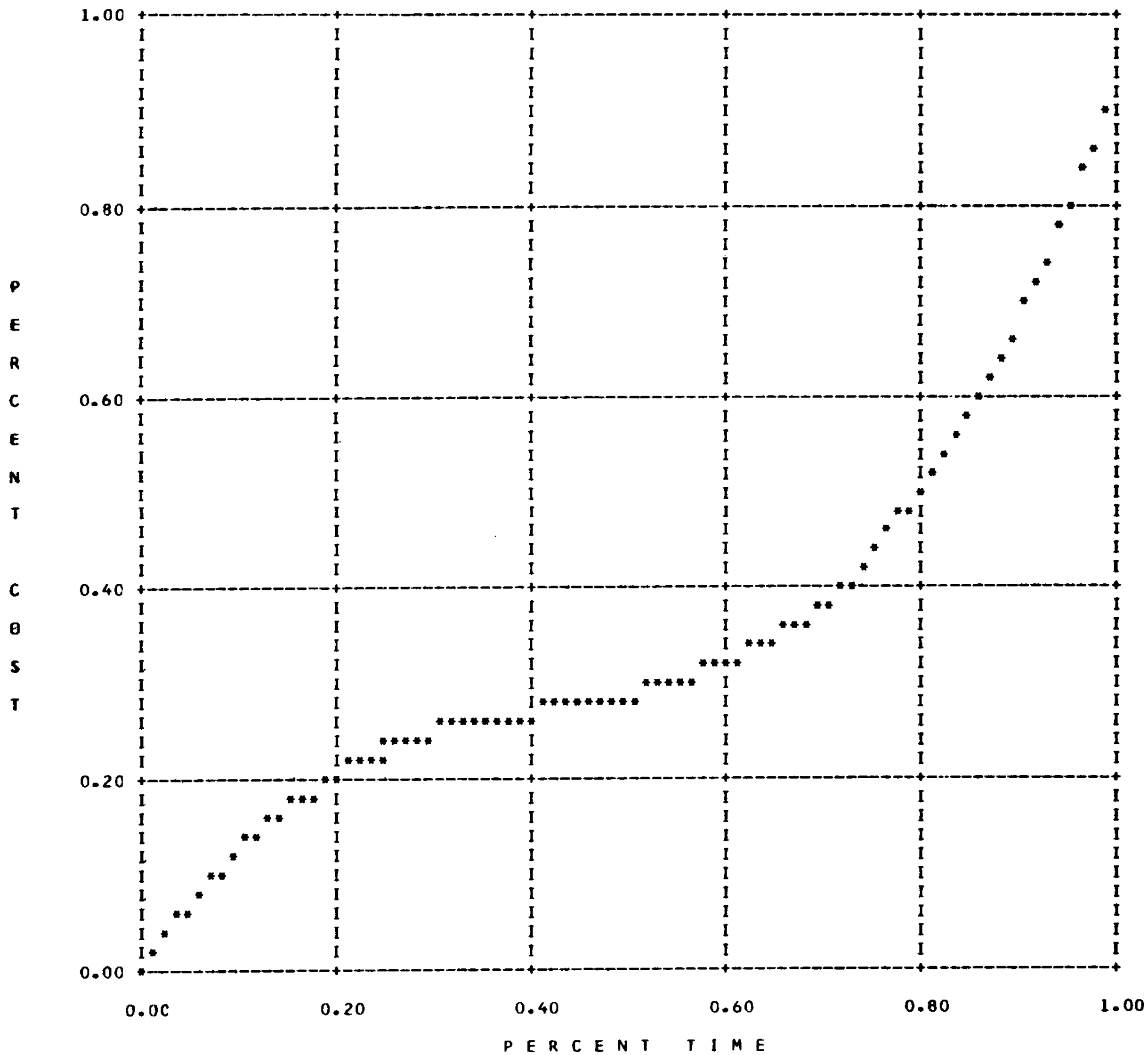
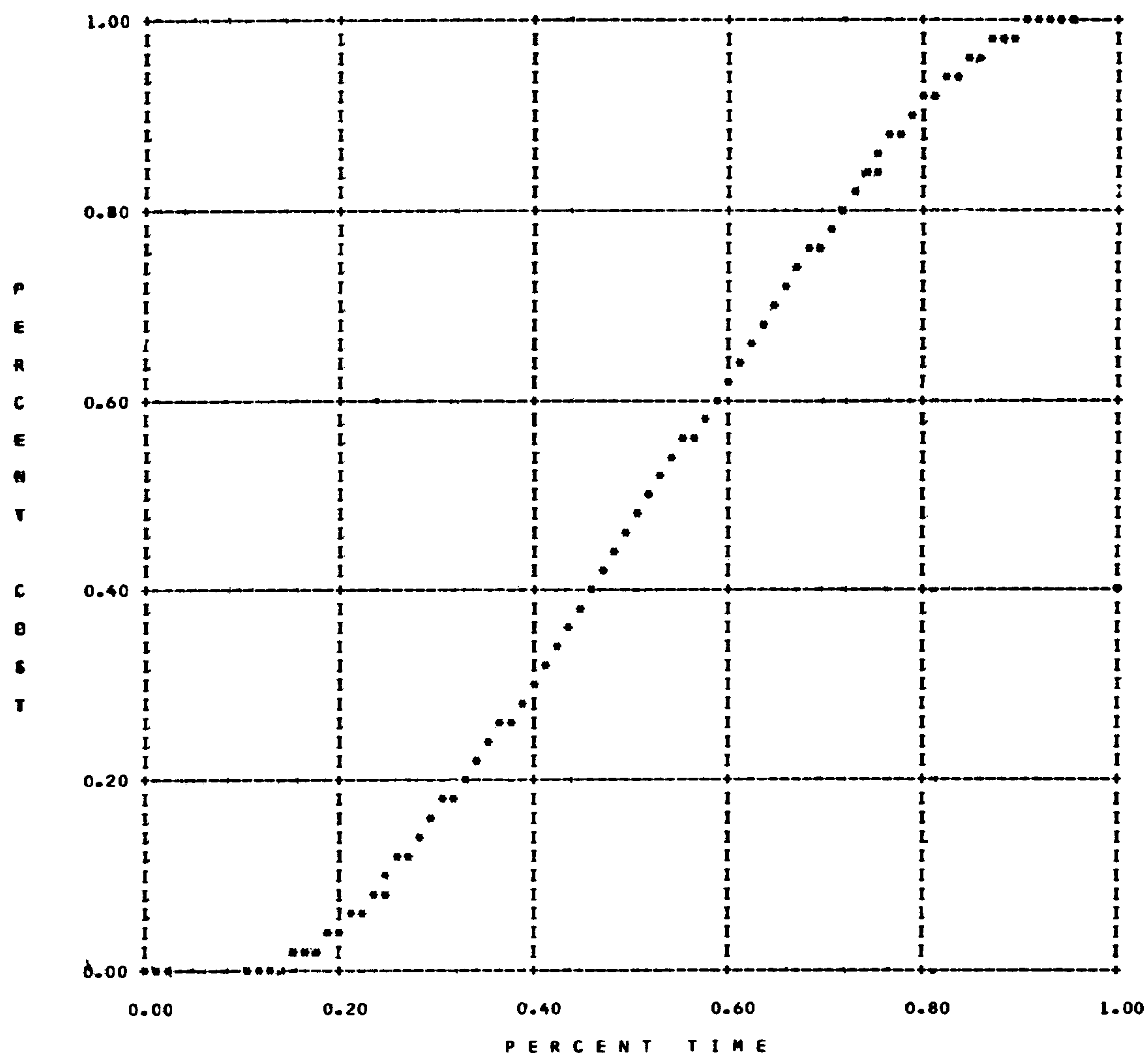


SELECTION SET 1 - CURVE NUMBER 7



SELECTION SET 1 - CURVE NUMBER 8



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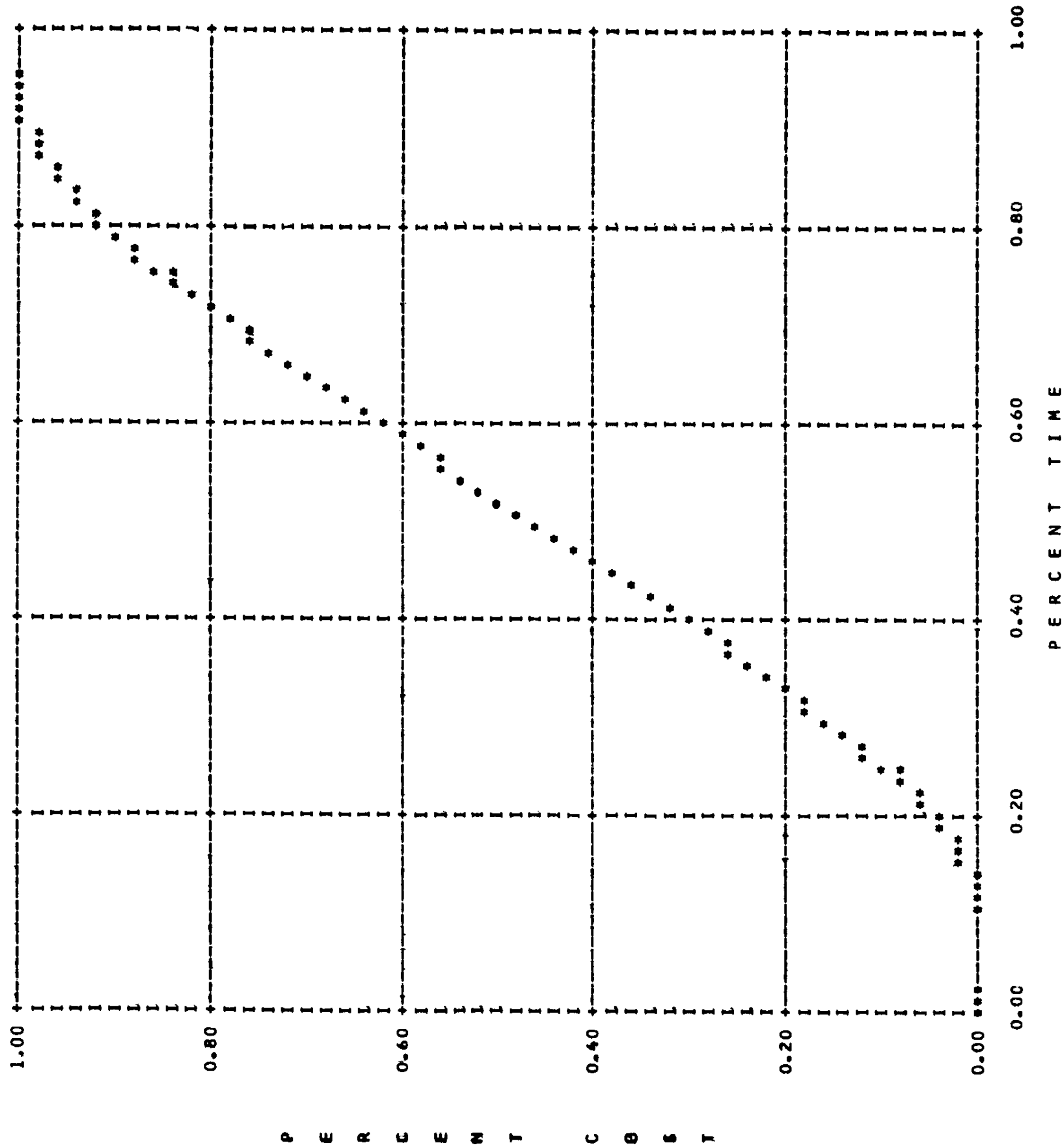
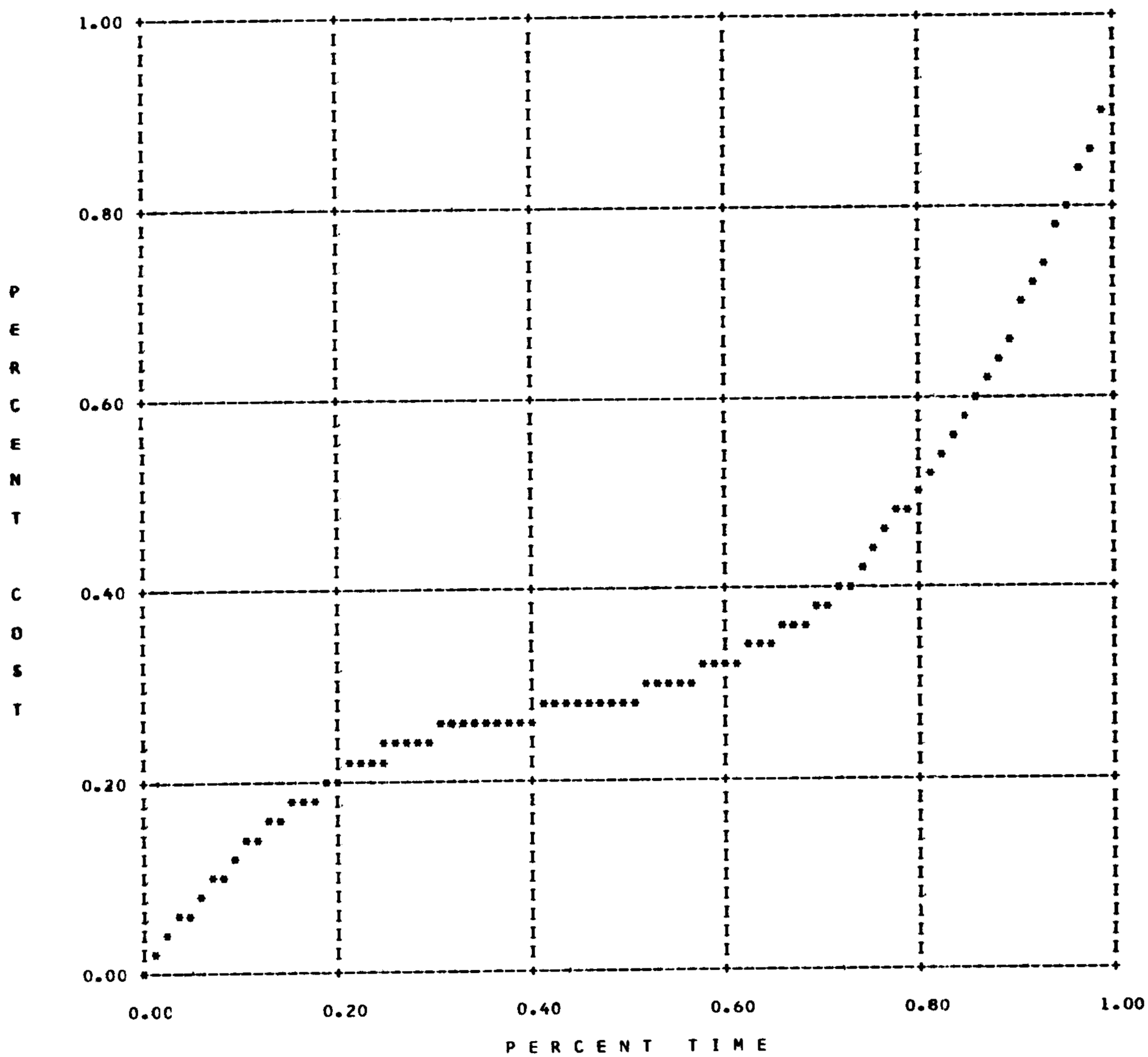


SELECTION SET 2 - CURVE NUMBER 1



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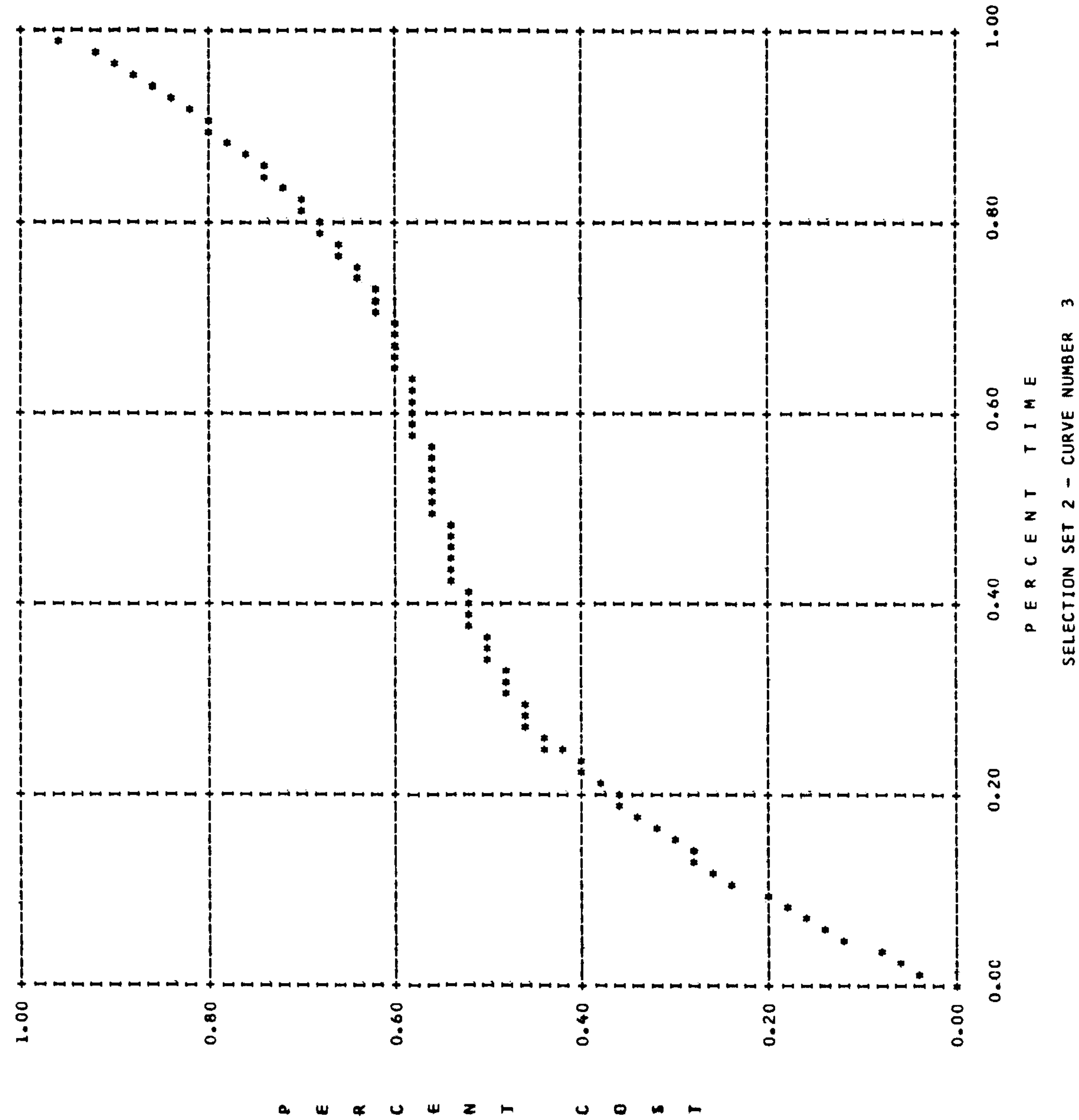
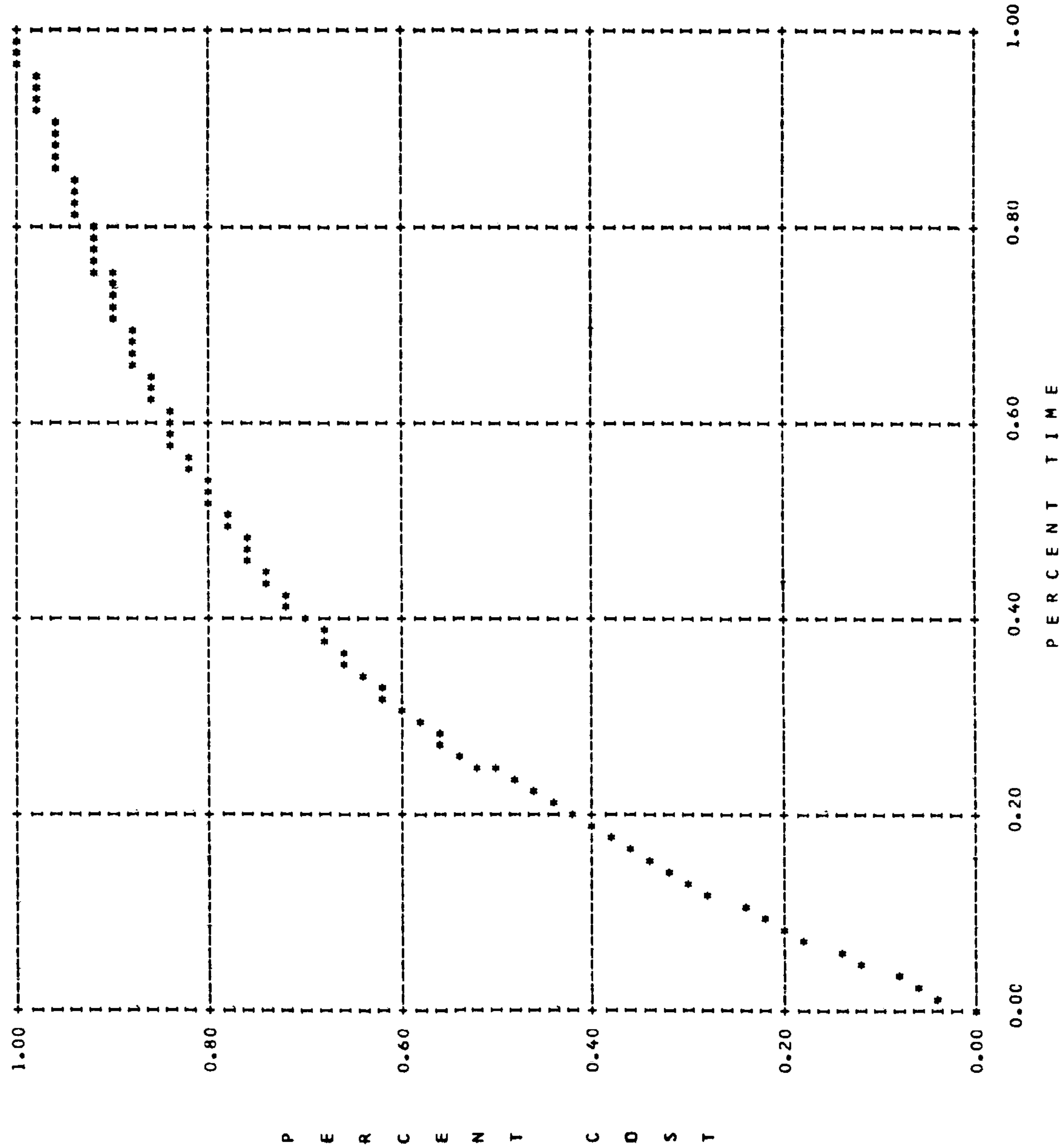

P E R C E N T   C O S T
1.00
0.80
0.60
0.40
0.20
0.00
0.0C
0.20
0.40
0.60
0.80
1.00
P E R C E N T   T I M E
SELECTION SET 2 - CURVE NUMBER 3



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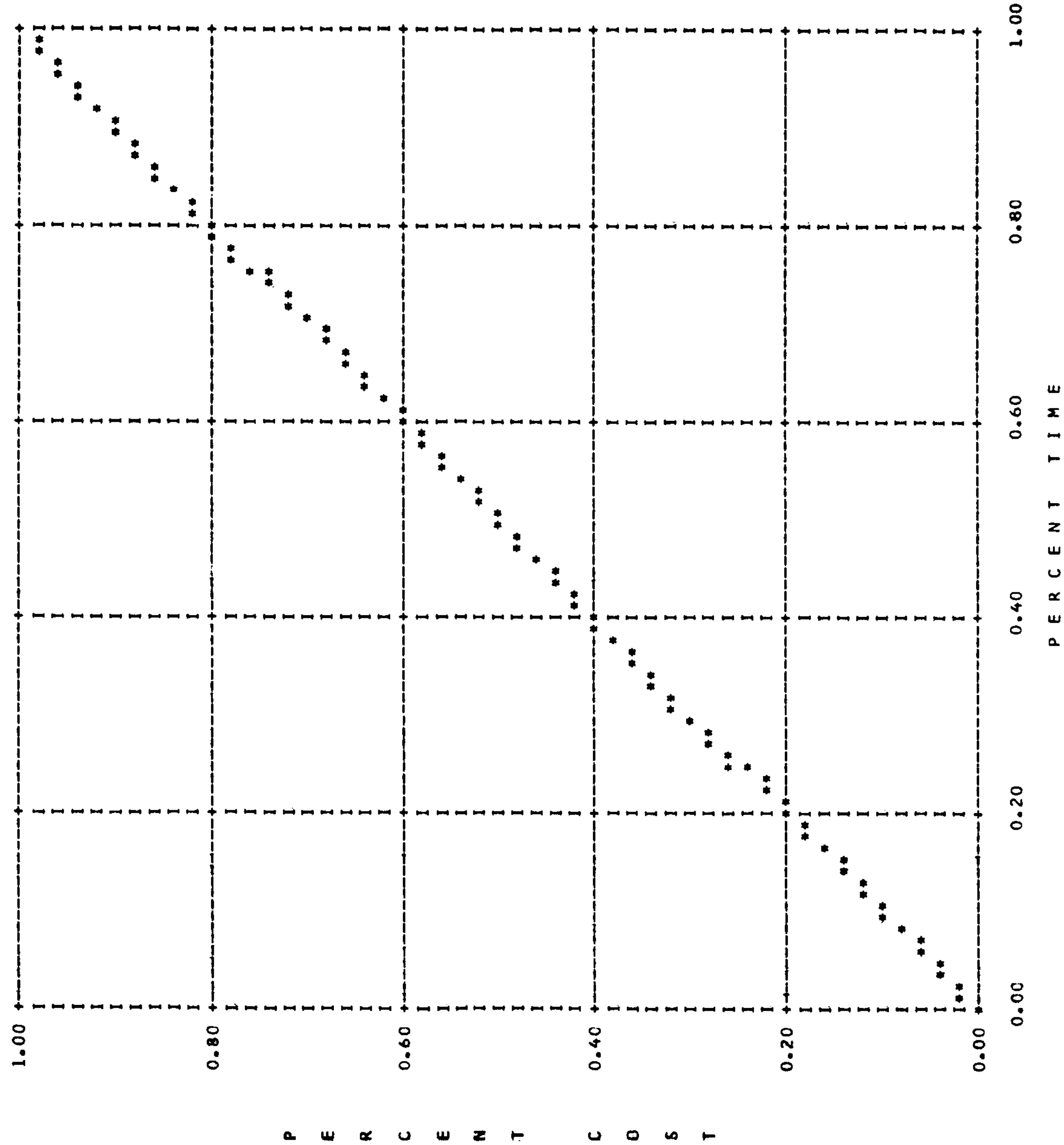
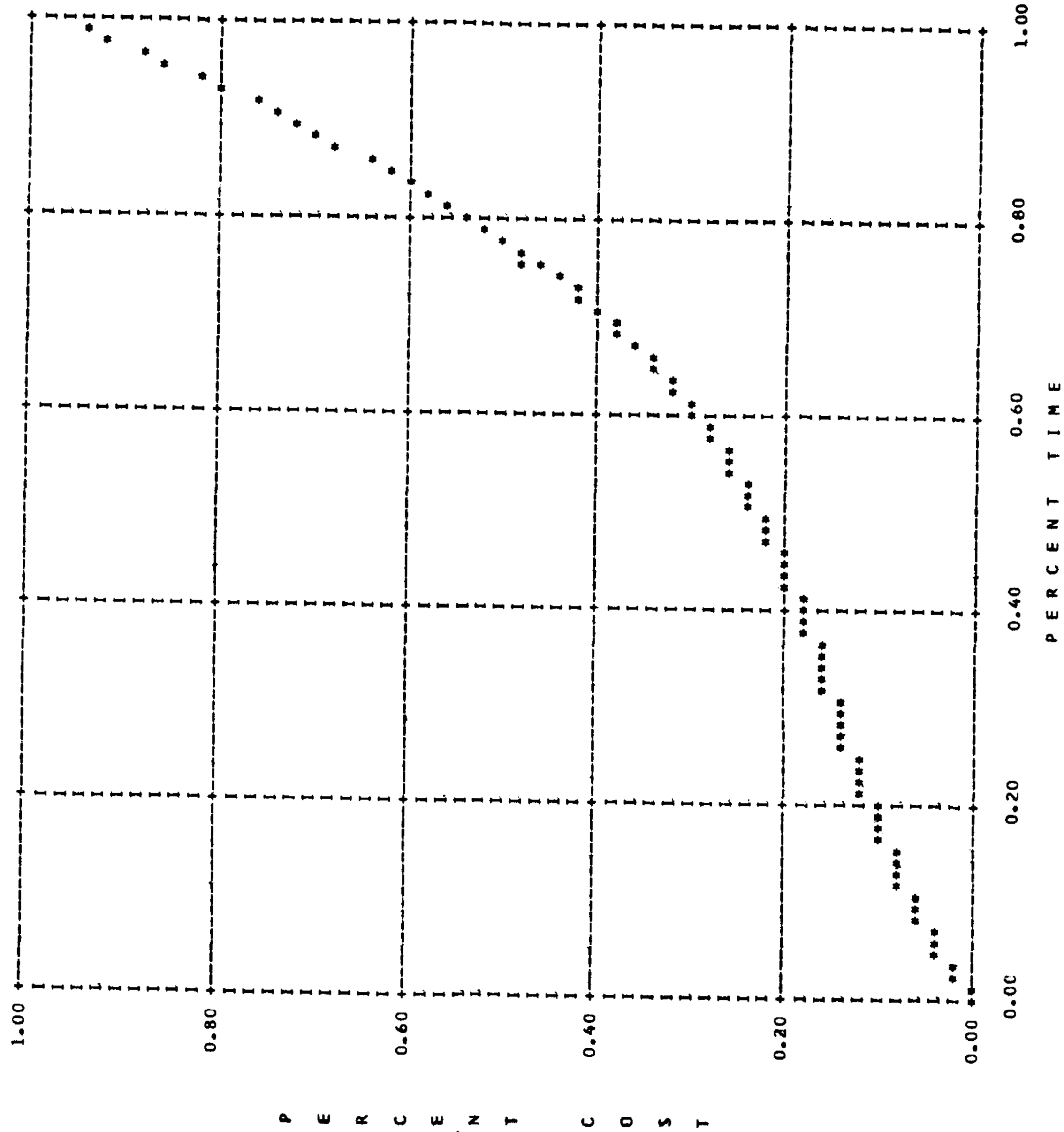
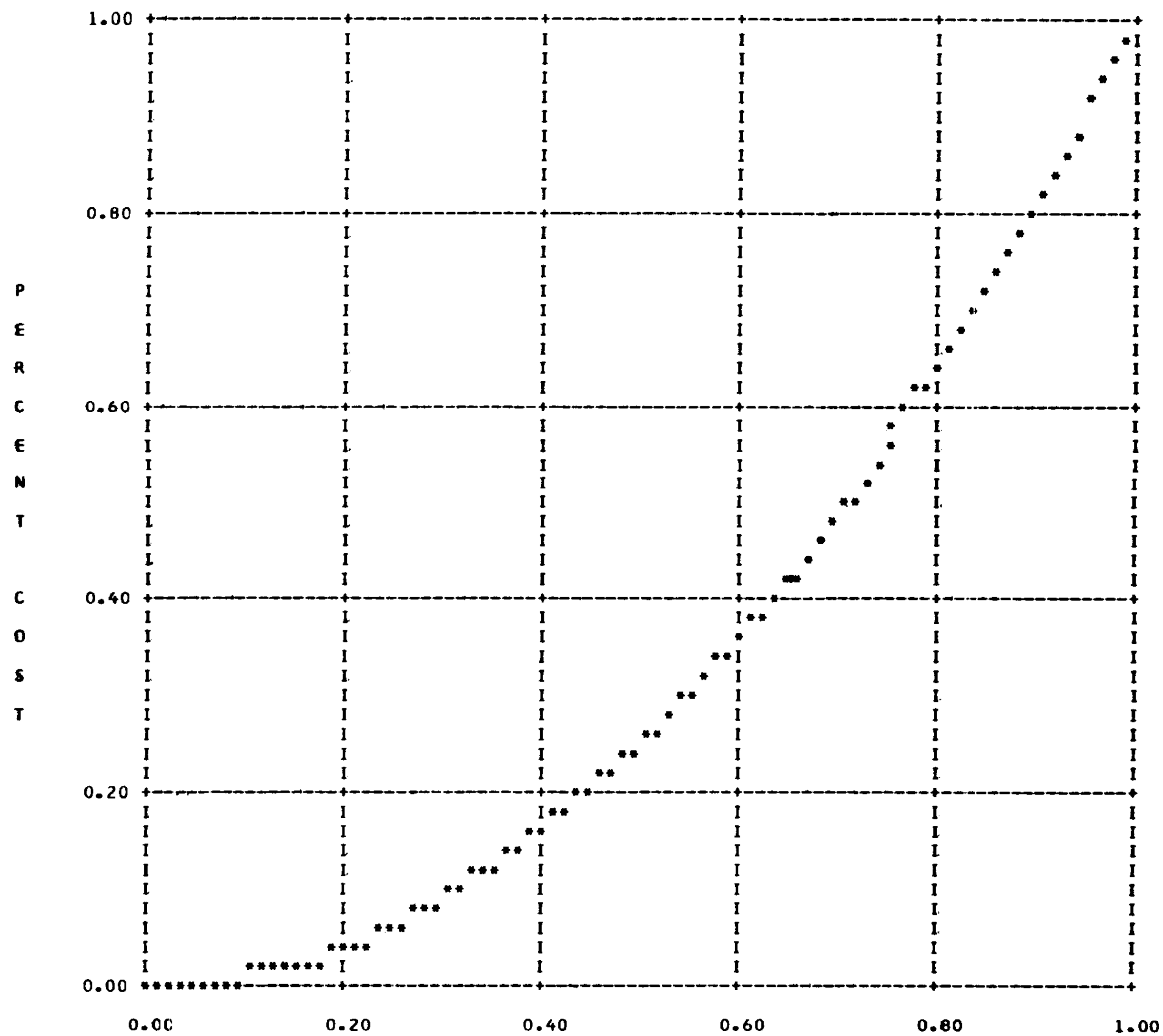


SELECTION SET 2 - CURVE NUMBER 5



SELECTION SET 2 - CURVE NUMBER 6



SELECTION SET 2 - CURVE NUMBER 7

HISTORY - PROBLEM 1

NUMBER OF CURVES IN SELECTION SET = 9

NUMBER OF JUDGES = 10

VOTES

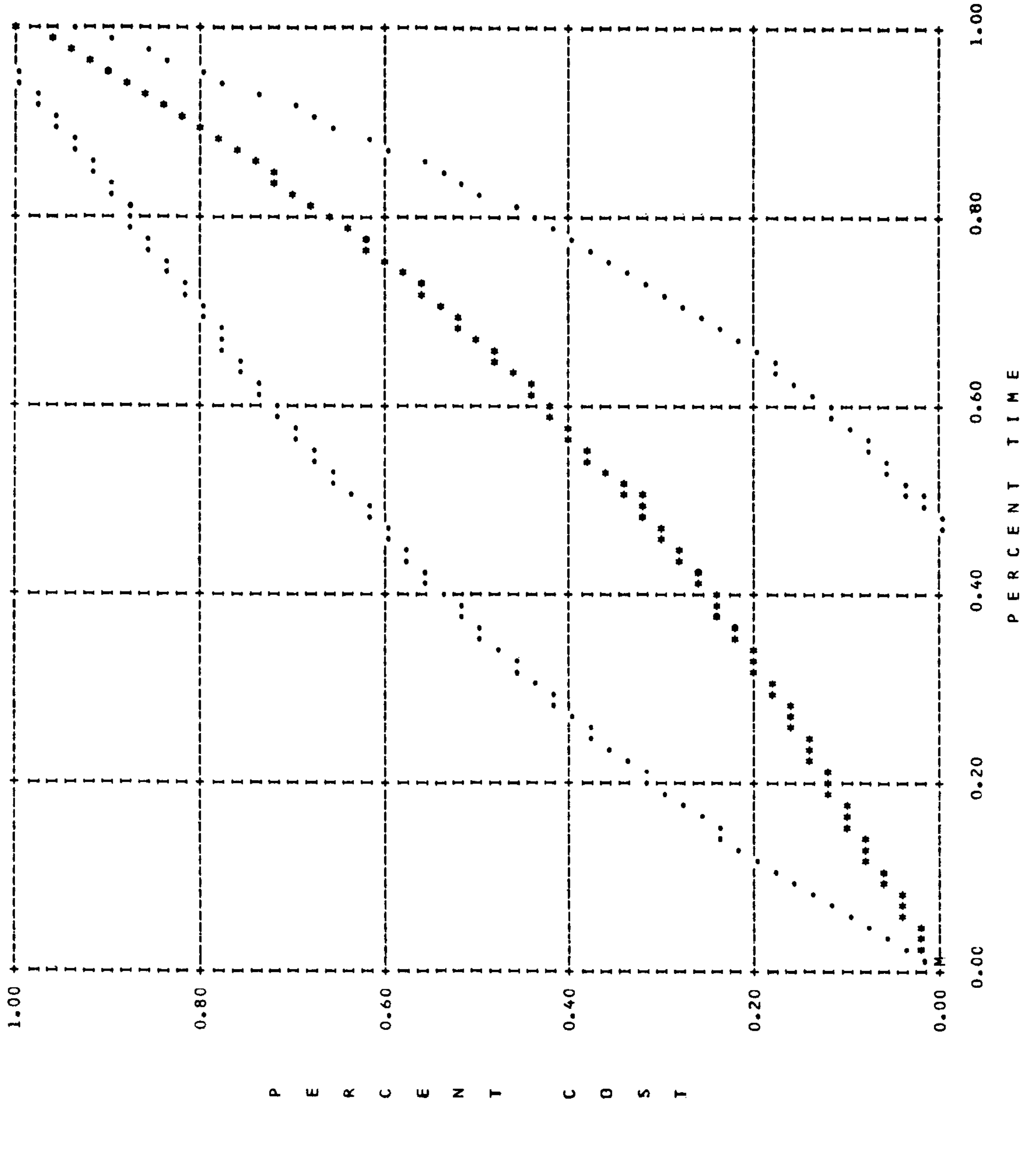
JUDGE	1	1
JUDGE	2	3
JUDGE	3	5
JUDGE	4	7
JUDGE	5	1
JUDGE	6	3
JUDGE	7	1
JUDGE	8	1
JUDGE	9	1
JUDGE	10	3

PROBABILITY OF OCCURRENCE = 0.00136

ANALYSIS CONTINUES

COEFFICIENTS OF FITTED POLYNOMIALS

	X**3	X**2	X	C
AVERAGED FUNCTION	0.58376E 00	-0.21396E 00	0.62064E 00	0.
UPPER CONFIDENCE LIMIT	0.67660E 00	-0.14754E 01	0.18449E 01	0.
LOWER CONFIDENCE LIMIT	0.49092E 00	0.10475E 01	-0.60360E 00	0.



PERCENT TIME
PERCENT COST
0.00
0.20
0.40
0.60
0.80
1.00

HISTORY - PROBLEM 1

NUMBER OF CURVES IN SELECTION SET = 9

NUMBER OF JUDGES = 10

VOTES

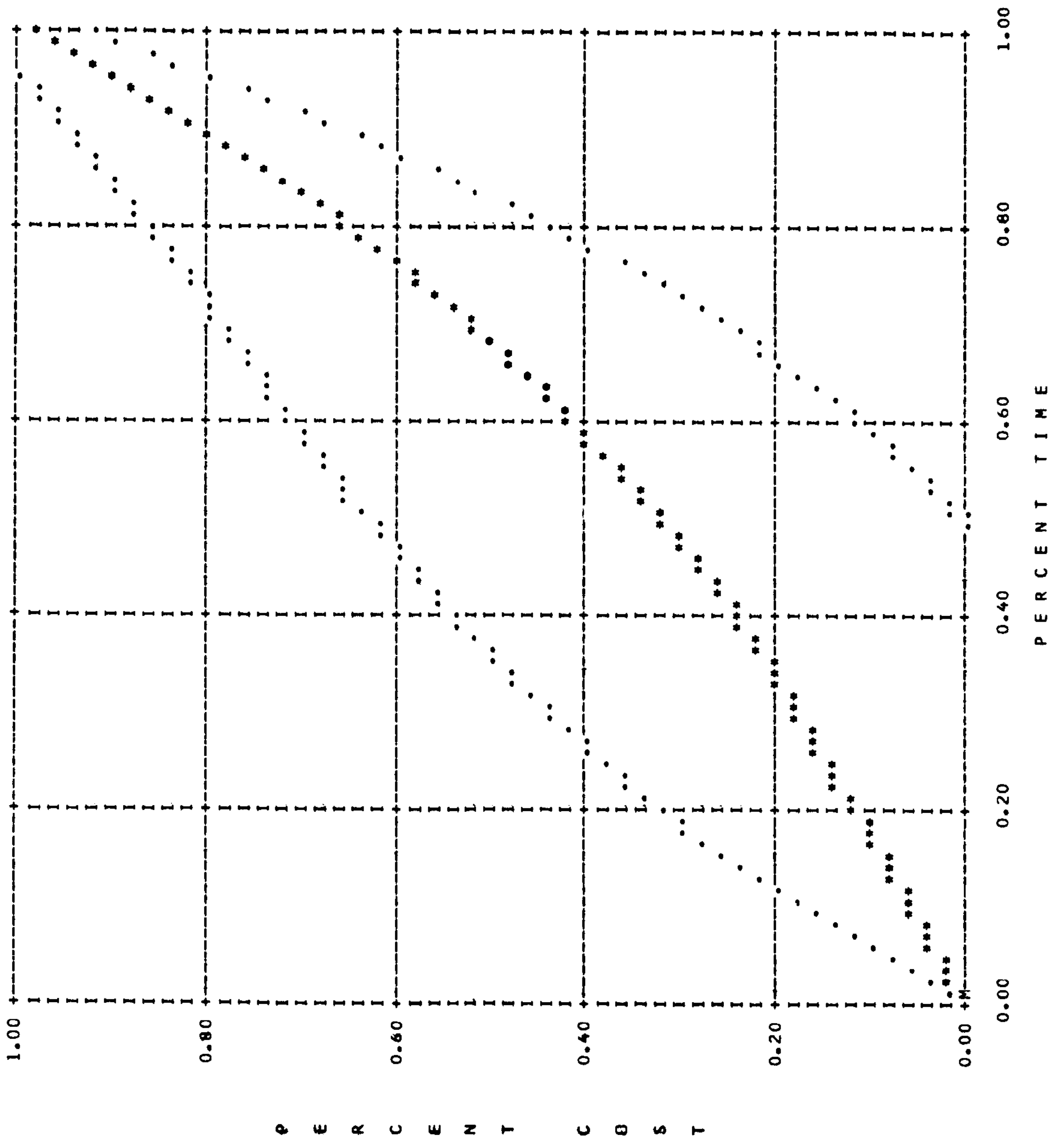
JUDGE	1	1
JUDGE	2	7
JUDGE	3	5
JUDGE	4	7
JUDGE	5	1
JUDGE	6	7
JUDGE	7	1
JUDGE	8	1
JUDGE	9	1
JUDGE	10	3

PROBABILITY OF OCCURRENCE = 0.00136

ANALYSIS CONTINUES

COEFFICIENTS OF FITTED POLYNOMIALS

	X**3	X**2	X	C
AVERAGED FUNCTION	0.66804E 00	-0.30190E 00	0.62330E 00	0.
UPPER CONFIDENCE LIMIT	0.96843E 00	-0.18846E 01	0.19652E 01	0.
LOWER CONFIDENCE LIMIT	0.36766E 00	0.12808E 01	-0.71858E 00	0.


PERCENT TIME
PERCENT COST
0.00
0.20
0.40
0.60
0.80
1.00

HISTORY - PROBLEM 1

NUMBER OF CURVES IN SELECTION SET = 9

NUMBER OF JUDGES = 10

VOTES

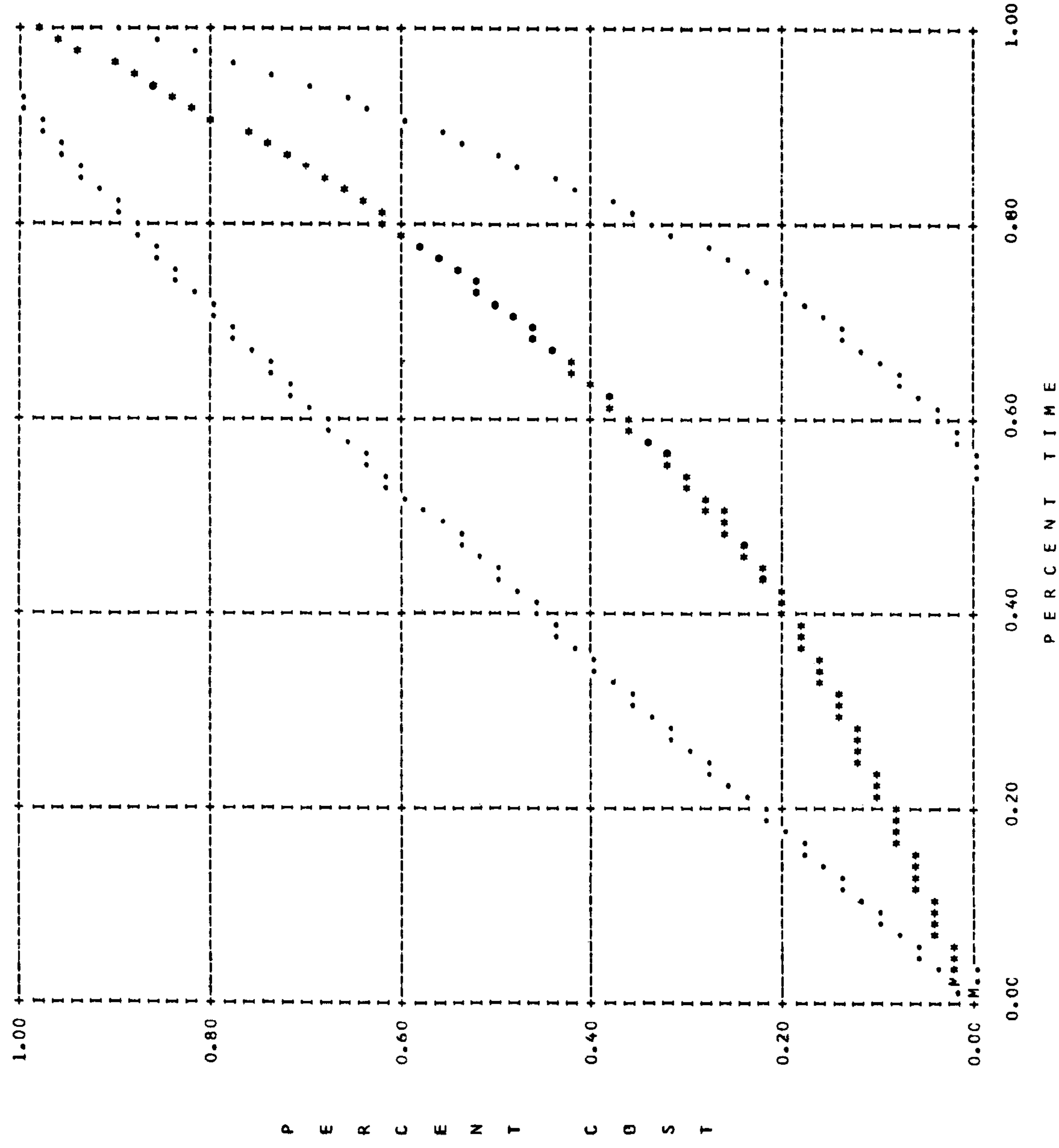
JUDGE	1	1
JUDGE	2	5
JUDGE	3	5
JUDGE	4	7
JUDGE	5	1
JUDGE	6	5
JUDGE	7	1
JUDGE	8	1
JUDGE	9	1
JUDGE	10	3

PROBABILITY OF OCCURRENCE = 0.00136

ANALYSIS CONTINUES

COEFFICIENTS OF FITTED POLYNOMIALS

	X**3	X**2	X	C
AVERAGED FUNCTION	0.74513E 00	-0.22692E 00	0.46429E 00	0.
UPPER CONFIDENCE LIMIT	-0.14578E 00	0.53488E-01	0.11569E 01	0.
LOWER CONFIDENCE LIMIT	0.16360E 01	-0.50733E 00	-0.22832E 00	0.


PERCENT COST
1.00
0.80
0.60
0.40
0.20
0.0C
PERCENT TIME
0.0C
0.20
0.40
0.60
0.80
1.00

HISTORY - PROBLEM 2

NUMBER OF CURVES IN SELECTION SET = 9

NUMBER OF JUDGES = 12

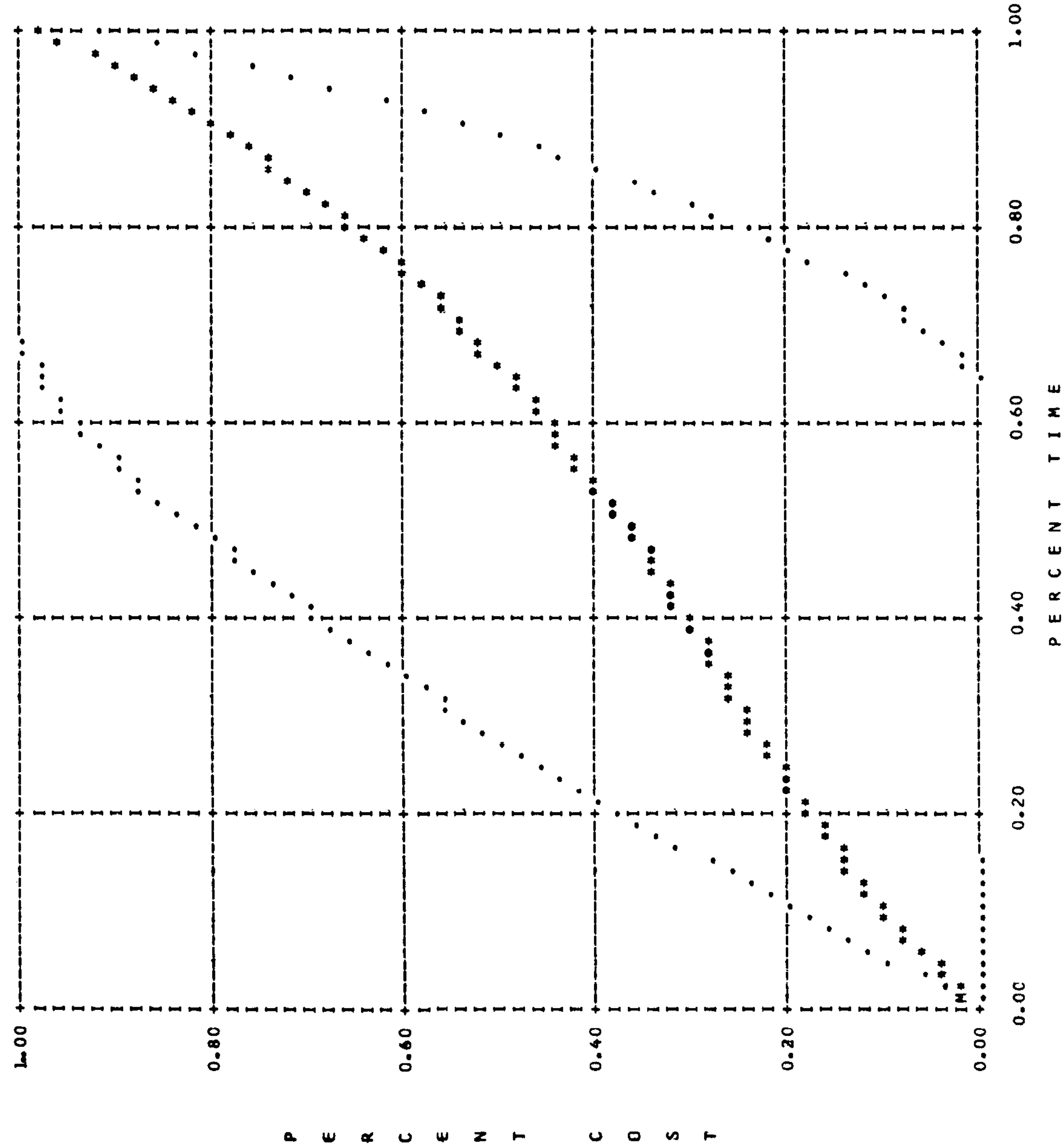
ORIGINAL VOTES		REVISED VOTES
9	JUDGE 1	9
9	JUDGE 2	9
4	JUDGE 3	4
6	JUDGE 4	6
8	JUDGE 5	8
2	JUDGE 6	2
2	JUDGE 7	2
8	JUDGE 8	8
9	JUDGE 9	9
6	JUDGE 10	6
4	JUDGE 11	4
1	JUDGE 12	0

PRCBABILITY OF OCCURRENCE = 0.01451

ANALYSIS CONTINUES

COEFFICIENTS OF FITTED POLYNOMIALS

	X**3	X**2	X	C
AVERAGED FUNCTION	0.11741E 01	-0.12972E 01	0.10982E 01	0.
UPPER CONFIDENCE LIMIT	-0.63177E 00	-0.30923E 00	0.19741E 01	0.
LOWER CONFIDENCE LIMIT	0.29800E 01	-0.22851E 01	0.22228E 00	0.


PERCENT TIME
0.00
0.20
0.40
0.60
0.80
1.00
PERCENT COST
0.00
0.20
0.40
0.60
0.80
1.00

HISTORY - PROBLEM 2

NUMBER OF CURVES IN SELECTION SET = 9

NUMBER OF JUDGES = 12

ORIGINAL VOTES		REVISED VOTES
9	JUDGE 1	0
9	JUDGE 2	9
4	JUDGE 3	0
6	JUDGE 4	6
8	JUDGE 5	8
2	JUDGE 6	2
2	JUDGE 7	2
8	JUDGE 8	8
9	JUDGE 9	9
6	JUDGE 10	6
4	JUDGE 11	0
1	JUDGE 12	0

NUMBER OF JUDGES REDUCED TO 8

ANALYSIS ABANDONED

HISTORY - PROBLEM 3

NUMBER OF CURVES IN SELECTION SET = 7

NUMBER OF JUDGES = 9

VOTES

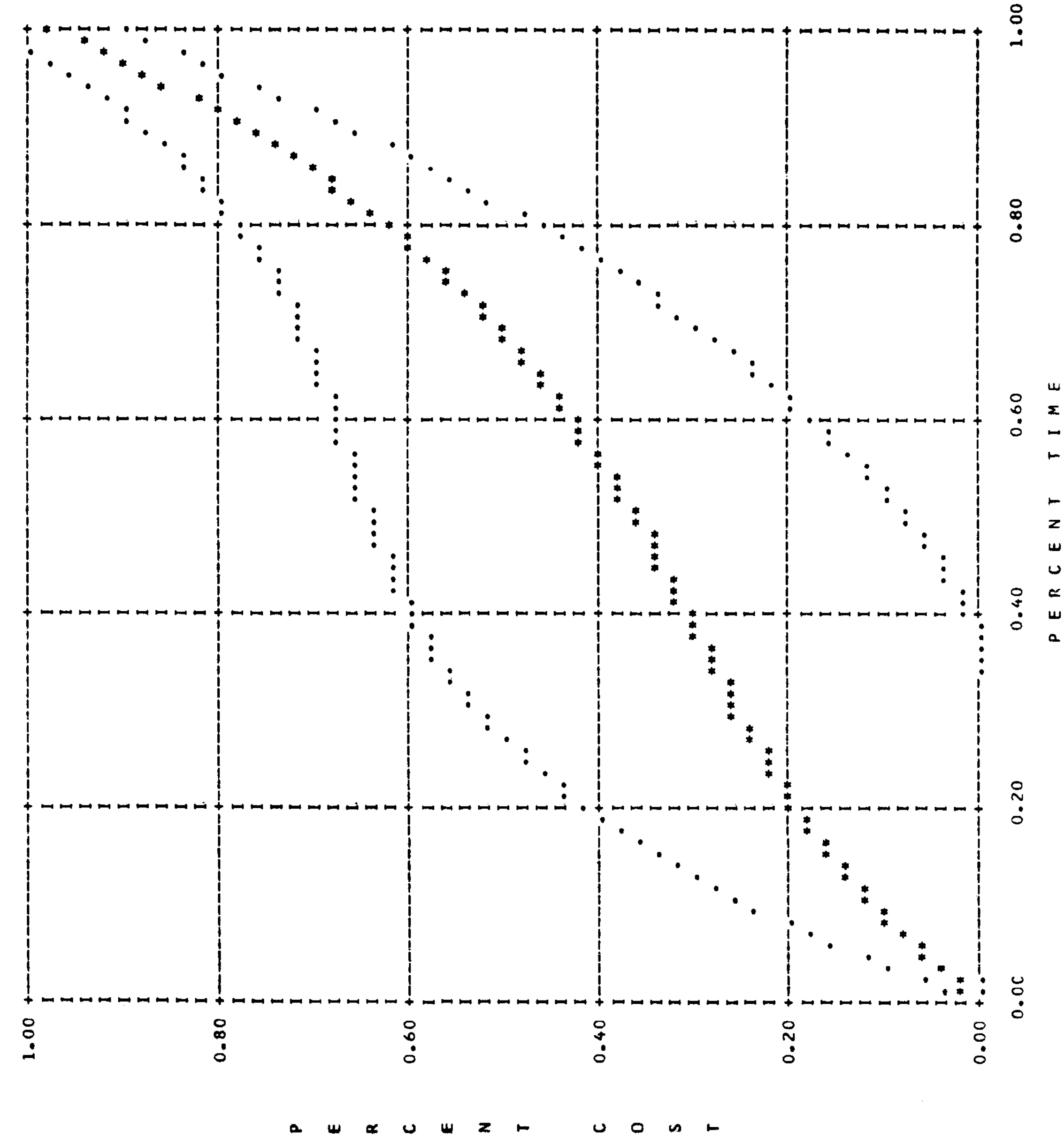
JUDGE	1	2
JUDGE	2	3
JUDGE	3	7
JUDGE	4	7
JUDGE	5	7
JUDGE	6	2
JUDGE	7	3
JUDGE	8	7
JUDGE	9	5

PROBABILITY OF OCCURRENCE = 0.01601

ANALYSIS CONTINUES

COEFFICIENTS OF FITTED POLONOMIALS

	X**3	X**2	X	C
AVERAGED FUNCTION	0.16237E 01	-0.19372E 01	0.12872E 01	0.
UPPER CONFIDENCE LIMIT	0.27239E 01	-0.45919E 01	0.29054E 01	0.
LOWER CONFIDENCE LIMIT	0.52345E 00	0.71743E 00	-0.33100E 00	0.


PERCENT TIME
0.0C
0.20
0.40
0.60
0.80
1.00
PERCENT COST
0.00
0.20
0.40
0.60
0.80
1.00

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